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Numerical computations of spaghetti fracture

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Abstract:

Peridynamics offers decisive advantages for the simulation of dynamic fracture and is used here to investigate the fracture of long beams with the material properties of dry spaghetti. The aim of this work is to identify boundary conditions that reproduce the experimentally known fracture behavior while remaining simple to parameterize and suitable for future studies. Different loading concepts for bending the beam up to fracture are therefore compared, including a four-point bending setup and simplified end-loading approaches. The simulations show that the four-point bending test can reproduce the expected fracture patterns, but involves many sensitive parameters. End loading by force density reduces the geometric complexity of the setup, yet the fracture behavior remains strongly dependent on the selected parameters and is difficult to realize experimentally. The most suitable alternative is a velocity-controlled load at the beam ends. In combination with a slightly pre-bent beam, it yields fracture patterns with similarly few fragments as the four-point bending test, keeps the simulation time sufficiently low, and allows the required simulation time to be estimated in advance. Thus, the developed velocity boundary condition provides a relatively easy-to-apply basis for future peridynamic investigations of spaghetti fracture.

Keywords: Peridynamics, Dynamic fracture, Nonlocal modeling

1 Introduction

In mechanical engineering, the damage and failure behavior of components is of great importance. Material damage is possible or even intended in practically every technical area, such as the collision of two cars, a bird strike on an aircraft or the demolition of a building [4]. In order to be able to make the best possible prediction of the damage behavior in advance, the component stress can be simulated using various methods. A commonly used method for simulating component behavior is the finite element method (FEM), but its use in the case of discontinuities in the continuum (e.g. due to cracks) is generally not possible or very time-consuming. A relatively new method compared to the FEM is peridynamics, which, in contrast to the FEM, is not based on displacement derivatives but on integro-differential equations and therefore also allows for discontinuities in the continuum.

The advantage of peridynamics over the FEM is particularly evident, for example, when simulating the deflection of a long beam until it breaks. Such calculations are very time-consuming to perform with other FEM-based simulation methods and have so far only been achievable with simplified two-dimensional models.

The aim of this work is to use peridynamics to simulate fractures of such long beams and to investigate the influence of various boundary conditions and parameters on the fracture behavior. Thus, an approach that closely approximates reality and is straightforward

to implement should be developed, suitable for future peridynamic studies.

Three different approaches or boundary conditions are used in this work to simulate the bending of the beam up to fracture. In the first approach, forces are applied to four areas on the lateral surfaces of the beam (four-point bending test). However, this approach includes many variables to be investigated, which complicates the analysis. In order to make it easier to compare the simulation results with the results obtained in possible subsequent experimental investigations, an attempt is made to find approaches that are easier to reproduce in reality. The bending of the beam in a tensile testing machine serves as a model for the two other approaches. For this purpose, either a force density or a speed is applied to the two ends of the beam.

The paper is structured as follows: Section 2 introduces the fundamentals of peridynamics. Section 3 describes the characteristic fracture behavior of spaghetti. Section 4 first presents the model used in the simulations and the required material and model parameters. It then defines and investigates different boundary conditions and discusses the resulting simulation outcomes. Finally, Section 5 summarizes the main findings and critically assesses the methods investigated.

2 Peridynamics theory

Peridynamics (from the Greek prefix “peri,” meaning “near”) is a nonlocal formulation of continuum mechanics introduced by Silling in 2000 [12]. In this formulation, the body $\mathcal{B}$ is described using a grid of material points. The formulation is therefore referred to as nonlocal because a material point of a body is also influenced by other material points (hereinafter abbreviated as MP) that are not its immediate neighbors, but lie within a defined circular or spherical region around the MP [5]. This region is called the horizon δ . All MP within the horizon are referred to as the point family $\mathcal{H}$, which is defined as

$$\mathcal{H}(\mathbf{X}) = \{\mathbf{X}' \in \mathcal{B}_0 \mid 0 < |\mathbf{X}' - \mathbf{X}| \leq \delta\}, \quad (1)$$

as can be seen in Figure 1 [12]. The classical model of continuum mechanics, on the other hand, is local, meaning that MPs interact only with their immediate neighbors [5].

The position of an MP is specified by its coordinate vector $\mathbf{X}$, and its displacement by its displacement vector $\mathbf{U}$ (see Figure 1). The interaction between an MP $\mathbf{X}$ and one of its neighboring MPs $\mathbf{X}'$ is referred to as a bond $\Delta\mathbf{X} = \mathbf{X}' - \mathbf{X}$. This vector specifies the relative

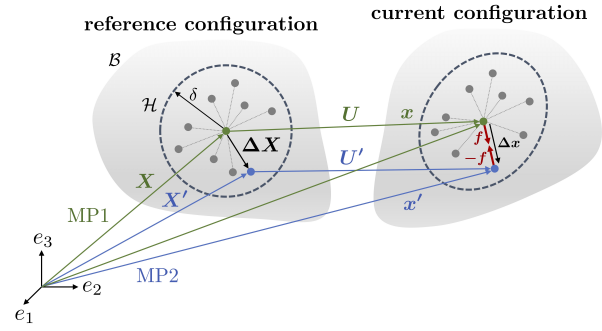


Figure 1 – Deformation of a point family in bond-based peridynamics [5, S.37]

position of the bond in the reference state or the undeformed state. When the body deforms, the MP $\mathbf{X}'$ shifts relative to $\mathbf{X}$, so the bond changes to $\Delta\mathbf{x} = \mathbf{x}' - \mathbf{x}$. The vector $\Delta\mathbf{U} = \mathbf{U}' - \mathbf{U}$ describes this bond displacement, where $\Delta\mathbf{x} = \Delta\mathbf{X} + \Delta\mathbf{U}$ holds [12].

In continuum mechanics, the equation of motion for a point is generally defined as

$$\rho \ddot{\mathbf{U}}(t, \mathbf{X}) = \bar{\mathbf{B}}_{\text{int}}(t, \mathbf{X}) + \bar{\mathbf{B}}_{\text{ext}}(t, \mathbf{X}), \quad (2)$$

where $\ddot{\mathbf{U}}$ is the acceleration vector, ρ is the density, $\bar{\mathbf{B}}_{\text{int}}$ is the internal force density (describing the internal forces per unit volume), and $\bar{\mathbf{B}}_{\text{ext}}$ is the external force density (describing the external forces per unit volume).

Classical continuum mechanics differs from peridynamics in the way it calculates the internal force density vector $\bar{\mathbf{B}}_{\text{int}}$. In classical continuum mechanics, the following holds:

$$\bar{\mathbf{B}}_{\text{int}}(t, \mathbf{X}) = \nabla \cdot \mathbf{P}(t, \mathbf{X}), \quad (3)$$

where $\mathbf{P}$ is the first Piola-Kirchhoff stress tensor. Displacement derivatives are therefore used for the calculation [14]. Since discontinuities (e.g., cracks) in the continuum result in discontinuities at these locations, meaning that the derivatives are not defined there, modeling using such models can be problematic.

In contrast, peridynamics uses an integro-differential equation that can also be applied to discontinuities to calculate the internal force density [5]. The internal force density vector can be calculated in various ways, the most important of which are presented below.

2.1 Bond-based (BB) peridynamics

Bond-based (BB) peridynamics is the first fully developed formulation of peridynamics [12]. This formulation is based on the principle that each bond acts independently of all other bonds [17]. Bonds break when their strain exceeds a critical value [17].

The internal force density is then defined as

$$\bar{\mathbf{B}}_{\text{int}}(t, \mathbf{X}) = \int_{\mathcal{H}(\mathbf{X})} \mathbf{f}(t, \mathbf{X}, \mathbf{X}') dV', \quad (4)$$

where $\mathbf{f}$ is defined as a pairwise force function and contains all the constitutive information about the material [12, 11]. It can be decomposed into a scalar component and a direction vector:

$$\mathbf{f}(t, \mathbf{X}, \mathbf{X}') = \tilde{f}(t, \mathbf{X}, \mathbf{X}') \frac{\Delta \mathbf{x}}{|\Delta \mathbf{x}|}. \quad (5)$$

The scalar part $\tilde{f}$ can be expressed using the formula [13]

$$\tilde{f}(t, \mathbf{X}, \mathbf{X}') = \mu \cdot C_b \cdot \varepsilon_b(t, \mathbf{X}, \mathbf{X}'), \quad (6)$$

where ε_b denotes the bond strain, C_b denotes the bond constant, and μ denotes the damage parameter. According to Newton's third law, the pairwise force function must also satisfy [12]:

$$\mathbf{f}(t, \mathbf{X}, \mathbf{X}') = -\mathbf{f}(t, \mathbf{X}', \mathbf{X}), \quad (7)$$

Thus, $\mathbf{f}$ is directed in opposite directions for a material point and its neighbor and is equal in magnitude (see Figure 1).

The bond strain ε_b is defined as [13]

$$\varepsilon_b(t, \mathbf{X}, \mathbf{X}') = \frac{|\Delta \mathbf{x}| - |\Delta \mathbf{X}|}{|\Delta \mathbf{X}|} = \frac{|\Delta \mathbf{x}|}{|\Delta \mathbf{X}|} - 1. \quad (8)$$

Once the bond strain reaches a critical value, the so-called critical strain ε_0 , the bond breaks irreversibly [17]. The critical strain ε_0 is defined as [13]

$$\varepsilon_0 = \sqrt{\frac{10 \cdot G_c}{\pi \cdot C_b \cdot \delta^5}}, \quad (9)$$

where the so-called energy release rate G_c is the work required to break all bonds per unit fracture surface area. The bond constant C_b is calculated with

$$C_b = \frac{18K}{\pi \delta^4}, \quad (10)$$

where K is the bulk modulus [13]. The damage parameter μ is defined as follows:

$$\mu = \begin{cases} 1, & \text{if } \varepsilon_b < \varepsilon_0 \\ 0, & \text{otherwise} \end{cases}, \quad (11)$$

thus assumes the value 0 when the bond is broken and the value 1 when the bond is intact [13].

The bond-based formulation, however, has some limitations [15]: Since each material point pair interacts

independently of other conditions, only a Poisson's ratio of

$$\nu = \begin{cases} \frac{1}{3}, & \text{if plane stress state} \\ \frac{1}{4}, & \text{if 3D or plane strain state} \end{cases} \quad (12)$$

is possible [18]. Furthermore, the forces in the bonds cannot be represented using the conventional strain and stress tensors from continuum mechanics. Since only a distinction is made between intact and broken bonds, no distinction can be made between elastic and plastic deformation either [15].

2.2 State-based (SB) peridynamics

A generalization of peridynamic theory is provided by state-based (SB) peridynamics, which resolves the aforementioned problems. It was introduced by Silling and is a more general formulation overcoming the limitations of bond-based peridynamics [15, 17]. The internal force density thus depends not only on all neighbors of an MP, but also on the averaged state of all neighbors [6]. For this purpose, the so-called deformation state or force vector state $\mathbf{t}$ is defined [15], which maps the deformation state at all points within an influence volume into a force state [2].

The internal force density in state-based peridynamics is calculated using the formula

$$\bar{\mathbf{B}}_{\text{int}}(t, \mathbf{X}) = \int_{\mathcal{H}(\mathbf{X})} \mathbf{t}(t, \mathbf{X}, \mathbf{X}') - \mathbf{t}(t, \mathbf{X}', \mathbf{X}) dV', \quad (13)$$

where $\mathbf{t}$ is referred to as the force vector state [15]. A distinction is made here between ordinary state-based (OSB) and non-ordinary state-based (NOSB) peridynamics. While in OSB peridynamics the force vector states are parallel (Figure 2), the force vector states in NOSB peridynamics are not parallel (Figure 3). The magnitudes of the force vector states can differ in both formulations. In the BB formulation, on the other hand, the force vector states are parallel and equal in magnitude (see Figure 1). In particular, NOSB peridynamics offers many advantages through the representability of bonds with conventional strain and stress tensors according to [17]:

- General material models can be applied.
- Stresses can be transmitted across bonds in all directions, enabling a more realistic modeling of the continuum.
- Nonlinear anisotropic materials can also be modeled.

2.2.1 Ordinary state-based (OSB)

In the case of OSB peridynamics, the force state $\mathbf{t}$ is first decomposed into a scalar part and a direction vector

[15]:

$$\mathbf{t}(t, \mathbf{X}, \mathbf{X}') = \tilde{t}(t, \mathbf{X}, \mathbf{X}') \cdot \frac{\Delta \mathbf{x}}{|\Delta \mathbf{x}|}. \quad (14)$$

The scalar state $\tilde{t}$ represents the magnitude of the force [19] and is split into a volumetric and a deviatoric part

$$\tilde{t}(t, \mathbf{X}, \mathbf{X}') = \tilde{t}_{\text{vol}}(t, \mathbf{X}, \mathbf{X}') + \tilde{t}_{\text{dev}}(t, \mathbf{X}, \mathbf{X}'). \quad (15)$$

Furthermore, the extension scalar state e_b is defined, which describes the change in length of a bond due to deformation and is likewise decomposed into a volumetric and a deviatoric part [15]:

$$e_b(t, \mathbf{X}, \mathbf{X}') = |\Delta \mathbf{x}| - |\Delta \mathbf{X}| \quad (16)$$

$$= e_b^{\text{vol}}(t, \mathbf{X}, \mathbf{X}') + e_b^{\text{dev}}(t, \mathbf{X}, \mathbf{X}'). \quad (17)$$

The deviatoric part $\tilde{t}_{\text{dev}}$ is defined according to [15] as

$$\tilde{t}_{\text{dev}}(t, \mathbf{X}, \mathbf{X}') = \frac{\partial W}{\partial e_b^{\text{dev}}(t, \mathbf{X}, \mathbf{X}')} \quad (18)$$

with the deformation energy density W . The volumetric part $\tilde{t}_{\text{vol}}$ is calculated as

$$\tilde{t}_{\text{vol}}(t, \mathbf{X}, \mathbf{X}') = \frac{-3p}{m(\mathbf{X})} \cdot \omega(\Delta \mathbf{X}) \cdot |\Delta \mathbf{X}| \quad (19)$$

with the peridynamic pressure p , the influence function ω , and the weighted volume m . The peridynamic pressure is calculated according to Silling [15] as

$$p = -\frac{\partial W}{\partial \theta(t, \mathbf{X})} \quad (20)$$

and the weighted volume as

$$m(\mathbf{X}) = \int_{\mathcal{H}(\mathbf{X})} \omega(\Delta \mathbf{X}) \cdot |\Delta \mathbf{X}|^2 dV. \quad (21)$$

For a linear peridynamic fluid given by

$$W = \frac{K \cdot \theta(t, \mathbf{X})^2}{2} \quad (22)$$

where K is the bulk modulus and the dilatation θ is calculated as

$$\theta(t, \mathbf{X}) = \frac{3}{m(\mathbf{X})} \int_{\mathcal{H}(\mathbf{X})} \omega(\Delta \mathbf{X}) \cdot |\Delta \mathbf{X}| \cdot e_b(t, \mathbf{X}, \mathbf{X}') dV, \quad (23)$$

$\tilde{t}$ is then obtained as

$$\tilde{t}(t, \mathbf{X}, \mathbf{X}') = \frac{3K \cdot \theta(t, \mathbf{X})}{m(\mathbf{X})} \omega(\Delta \mathbf{X}) \cdot |\Delta \mathbf{X}|. \quad (24)$$

For a linear peridynamic solid for which the deformation energy density is given by

$$W = \frac{K \cdot \theta^2}{2} + \frac{\alpha}{2} \int_{\mathcal{H}} \omega \cdot e_b^{\text{dev}} \cdot e_b^{\text{dev}} dV, \quad (25)$$

as given in [15], $\tilde{t}$ is then obtained as

$$\tilde{t} = \frac{3K \cdot \theta}{m} \omega \cdot |\Delta \mathbf{X}| + \alpha \cdot \omega \cdot e_b^{\text{dev}}. \quad (26)$$

The constant α can be represented for the case of an isotropic solid with shear modulus G as [15]:

$$\alpha(\mathbf{X}) = \frac{15G}{m(\mathbf{X})}. \quad (27)$$

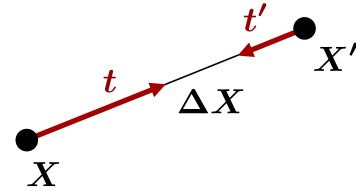


Figure 2 – OSB peridynamics: The force vector states are parallel and not equal in magnitude [15]

2.2.2 Non-ordinary state-based (NOSB)

In the case of NOSB peridynamics, the force state $\mathbf{t}$ is composed of the influence function ω , the Piola-Kirchhoff stress tensor $\mathbf{P}$, and the stiffness matrix $\mathbf{K}$ [6]:

$$\mathbf{t}(t, \mathbf{X}, \mathbf{X}') = \omega(\Delta \mathbf{X}) \cdot \mathbf{P}(t, \mathbf{X}) \cdot |\Delta \mathbf{X}| \cdot \mathbf{K}(\mathbf{X})^{-1}. \quad (28)$$

The influence function ω is defined as

$$\omega(\Delta \mathbf{X}) = 1 + \frac{\delta}{|\Delta \mathbf{X}|}, \quad (29)$$

thus having a higher value for MPs located closer to MP $\mathbf{X}$, giving them stronger weighting than MPs at greater distances.

The Piola-Kirchhoff stress tensor $\mathbf{P}$ is calculated from the Helmholtz free energy density ψ and the deformation gradient $\mathbf{F}$ [6]:

$$\mathbf{P}(t, \mathbf{X}) = \frac{\partial \psi}{\partial \mathbf{F}(t, \mathbf{X})}. \quad (30)$$

The stiffness matrix $\mathbf{K}$ is defined as

$$\mathbf{K}(\mathbf{X}) = \int_{\mathcal{H}(\mathbf{X})} \omega(\Delta \mathbf{X}) \cdot \Delta \mathbf{X} \otimes \Delta \mathbf{X} dV', \quad (31)$$

and the deformation gradient $\mathbf{F}$ as

$$\mathbf{F}(t, \mathbf{X}) = \left(\int_{\mathcal{H}(\mathbf{X})} \omega(\Delta \mathbf{X}) \cdot \Delta \mathbf{x} \otimes \Delta \mathbf{X} dV' \right) \mathbf{K}(\mathbf{X})^{-1}, \quad (32)$$

as defined in [6].

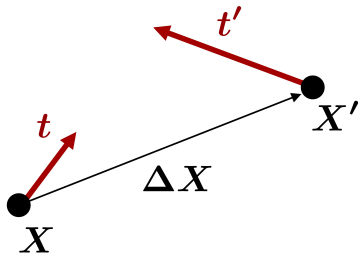
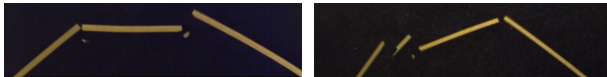


Figure 3 – NOSB peridynamics: The force vector states are not parallel and not equal in magnitude [15]



(a) Break into three fragments (b) Break into four fragments



(c) Break into five fragments

Figure 4 – Three spaghetti broken into different numbers of fragments, recorded with a high-speed camera [10]

3 The spaghetti fracture problem

Spaghetti was chosen as the model for the long beams because of its unique fracture behavior. If one holds a dry spaghetti by its ends and bend it in one direction beyond its load limit, it does not break into two parts as expected, but almost always into at least three parts (see Figure 4), as the physicist Richard Feynman already noted [3]. According to Audoly and Neukirch [1], the reason for this behavior lies in the formation of bending waves in the spaghetti. Accordingly, the first fracture occurs at the point where the curvature limit of the spaghetti is reached first. The sudden relaxation of the curvature at this point caused by the break leads to the release of bending waves. These in turn locally increase the curvature of the spaghetti and thus cause a second break. This mechanism can then repeat itself and cause further fractures.

However, if the spaghetti is not only bent, but also twisted, according to Heisser et al. [3], it should be able to break into just two fragments. To do this, the spaghetti must be twisted strongly (by at least 250°), as this reduces the curvature of the spaghetti before it breaks.

4 Peridynamic modeling of spaghetti fracture

In the following, the bending of a beam with different types of boundary conditions will be simulated and its fracture behavior analyzed. The material parameters of the beam were assumed as shown in Table 1 and correspond to those of a dry spaghetti as previously described [16, 3]. In this section, all quantities are reported in SI units: stress and modulus in GPa, force in N, length in m, and time in s.

parameters	symbol	value
density	ρ	1600 kg/m^3
Young's modulus	E	2.3 GPa
horizon	δ	$3.015 \times 10^{-3} \text{ m}$
Poisson's ratio	ν	0.25
critical bond strain	ε_0	0.013
MP-distances	Δx	$5 \times 10^{-4} \text{ m}$
length	l_x	0.2 m
width	l_z	$2 \times 10^{-3} \text{ m}$
height	l_y	$2 \times 10^{-3} \text{ m}$

Table 1 – Material and model parameters of the beam

Compared to the beam model used, which consists of $4 \times 4 \times 400$ points, a model with a shorter length is shown in Figure 5 for illustration purposes. The simulations are carried out with the Julia package `Peridynamics.jl` under the versions v0.2.0 to v0.3.3, where all methods are implemented [7, 8]. More specifically, the BB and OSB peridynamics, which are implemented in this package, were used. The BB peridynamics were used for the four-point bending test in subsection 4.1, and the OSB peridynamics were used for the other two approaches in subsection 4.2 and subsection 4.3. Both models produce results of similar quality and, given the specified parameters ($\nu = 1/4$ for all simulations), differ only in terms of the surface effect. Overall, it can be said that both models are equally suitable for use with spaghetti.

The choice of the horizon was based on previous work [9] and experience with peridynamic models. Its value influences crack initiation, crack path, and the number of fragments, particularly in BB and OSB peridynamics. However, since this work focuses primarily on comparing different boundary conditions, the influence of the horizon is negligible here.

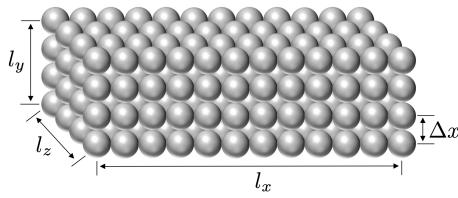


Figure 5 – Representation of the beam model (shortened in length l_x compared to the beam model used, consisting of $4 \times 4 \times 400$ MP)

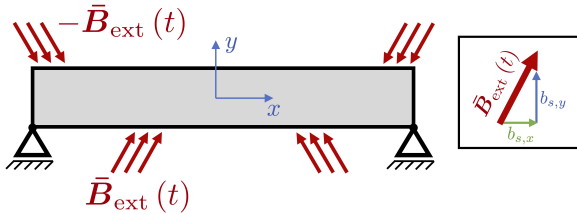


Figure 6 – Loading the ends of the beam with a force density function at four areas [10]

4.1 Four-point bending test

The boundary condition shown here represents the original approach to the simulation of beam breaks to be simplified in this work [10]. In this approach, the four areas of the beam shown in Figure 6 are subjected to a force density. The model used here is the classic breaking of a spaghetti with four fingers.

The external force density vector $\bar{\mathbf{B}}_{\text{ext}}$ can be formulated as the product of the magnitude force density b_s (unit N/m^3) and the direction vector $\mathbf{n}$ of the force density:

$$\bar{\mathbf{B}}_{\text{ext}} = b_s \cdot \mathbf{n}. \quad (33)$$

In this approach, the force density vector acts obliquely on the beam and is therefore divided into a force density $b_{s,y}$ acting in the y direction and a force density $b_{s,x}$ acting in the x direction. The curves of the force densities over time are shown in Figure 7. The force density $b_{s,y}$ increases linearly over time with

$$b_{s,y}(t) = \dot{b}_0 \cdot t, \quad (34)$$

where $\dot{b}_0$ indicates the gradient of the straight line [10]. The force density in the x -direction $b_{s,x}$ also increases linearly, but only with a 30% gradient compared to $b_{s,y}$ [10]:

$$b_{s,x}(t) = 0.3 \dot{b}_0 \cdot (t - t_x). \quad (35)$$

In addition, $b_{s,x}$ begins to take effect with a time delay at time t_x .

The following shows how a change in the gradient of the force densities $\dot{b}_0$ affects the number of beam

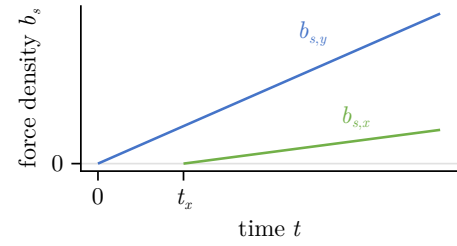


Figure 7 – Schematic curves of the force densities $b_{s,y}$ and $b_{s,x}$ over time [10]

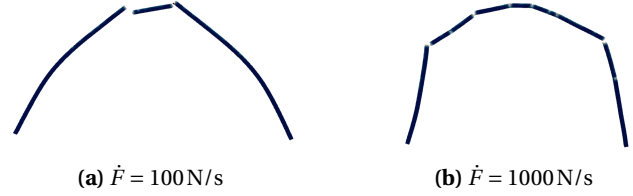


Figure 8 – Break the beam into three (a) and ten (b) fragments for different force densities

fragments created when the beam breaks and the time required until breakage t_B as well as the force F_B acting at this time. The simulations were carried out in a symmetrical and asymmetrical setting. The symmetric and asymmetric setups differ in the position of one of the four load areas: In the symmetric configuration, the load zones are arranged symmetrically about the center of the beam, whereas in the asymmetric configuration, the load zone located at the bottom right is shifted by $l_{yz} = 2 \times 10^{-3} \text{ m}$ toward the center of the beam.

Some of these simulation results are listed in Table 2 and are discussed below. For this approach, the force rate $\dot{F}$ (unit N/s) is given instead of the slope of the force density $\dot{b}_0$ (unit $\text{N/m}^3\text{s}$). When converting $\dot{b}_0$ to $\dot{F}$, it must be noted that $\dot{b}_0$ refers to the volume V_s , which comprises the outermost point layer of the beam end and is defined as follows:

$$V_s = \Delta x \cdot l_y \cdot l_z \quad (36)$$

$$= 2 \times 10^{-9} \text{ m}^3. \quad (37)$$

This allows the following relationship to be established for calculating the force rate $\dot{F}$:

$$\dot{F} = \dot{b}_0 \cdot V_s. \quad (38)$$

Basically, it can be deduced from the results that the greater the force rate $\dot{F}$ is selected, the more parts the beam breaks into, as also shown in Figure 8. In addition, the time until breakage is naturally shortened by a higher force density.

However, the range $\dot{F} = 500$ to 1300 N/s is striking, as the number of beam fragments does not increase or

$\dot{F}$ [N/s]	fragments		t_B [s]	F_B [N]
	sym.	asym.		
1300	8	11	0.0043	5.6
1000	8	10	0.0048	4.8
800	10	10	0.0055	4.4
500	7	7	0.0062	3.1
300	9	9	0.0075	2.3
200	6	7	0.0093	1.9
150	5	7	0.0108	1.6
100	3	3	0.0137	1.4

Table 2 – Variation of the force rate and its influence on the number of beam sections and breaking time

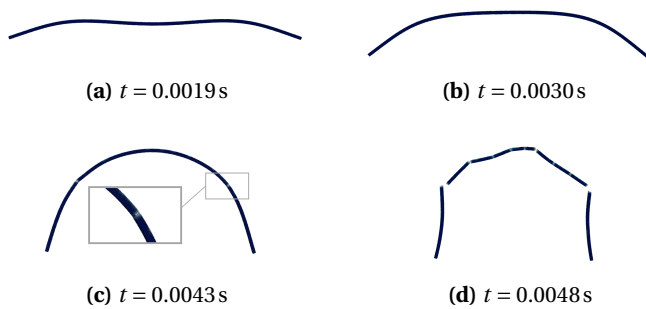


Figure 9 – Simulation progression of the beam for $\dot{F} = 1300 \text{ N/s}$

decrease as evenly as in the range $\dot{F} = 100$ to 300 N/s . This could be due to the fact that the beam is bent so quickly from a certain force rate that it first changes into a slight "M-shape", i.e. the middle of the beam bends slightly downwards (see Figure 9a). Only then does it bend completely in one direction (b) and break (d). As a result, the first break does not take place in the middle, as in the simulations with a smaller selected force density, but closer to the ends of the beam (c).

If the force rate is increased much further than $\dot{F} = 1300 \text{ N/s}$, the beam no longer bends completely in one direction, but also breaks in the "M-shape" (see Figure 10).

Using the respective force rate and the breaking time



Figure 10 – Break of the beam in the "M-shape" for $\dot{F} = 5000 \text{ N/s}$

in the formula

$$F_B = \dot{F} \cdot t_B \quad (39)$$

for the force acting on the beam at the time of breakage, it can be seen that the breaking force also decreases as the force rate decreases. This behavior could be due to the fact that F_B is a static variable, whereas the bending of the beam with subsequent fracture is a dynamic process.

In principle, the selected boundary condition is a suitable method for simulating such beam breaks. The speed at which the fracture occurs can be easily controlled by adjusting the force rate $\dot{F}$. The simulation time is also sufficiently short compared to the computational effort required. One problem, however, is the large number of different parameters (such as the size of the four areas on which the force density acts or the direction of the force density vector, etc.). This makes it difficult to set the parameter values in order to achieve simulation results that are as similar as possible to the experimentally determined fracture patterns. Furthermore, the simulation results are very sensitive to slight changes in the parameter values. Possible subsequent experimental checks of the results would therefore also be difficult. The boundary conditions presented in the following are intended to reduce the number of parameters and enable better experimental feasibility.

4.2 Load with cross-section-dependent force density function

The beam is now subjected to a force density at both ends $b_s(t)$ (see Figure 11).

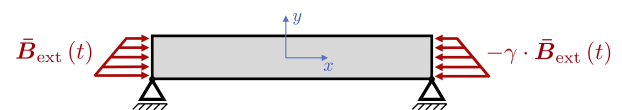


Figure 11 – Loading the ends of the beam with a force density function

In order to avoid a perfectly symmetrical break, the left and right ends of the beam are subjected to different force densities, as already described. For this purpose, the force density acting on the right end of the beam is multiplied by a factor, which is referred to below as γ (or "asymmetry factor"). Depending on the boundary condition, the value of γ is between 1.0001 and 1.1. A comparison of the effects of different values of γ on the beam simulations can be seen in Figure 12.

To ensure that the beam is not simply compressed, but that the beam buckles, the force density increases linearly across the cross-section. The two floating bearings

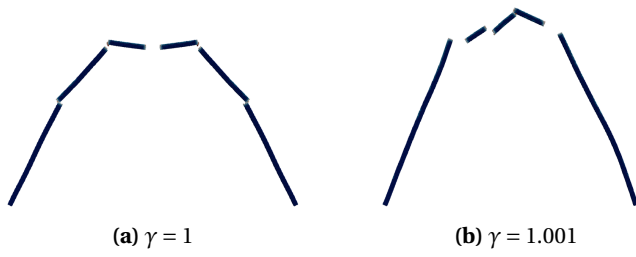


Figure 12 – Effect of a symmetrical (a) and an asymmetrical setting (b) on the fracture pattern (here the beam ends were subjected to a polynomial force density function)

ensure that the forces act exclusively in the x direction, analogous to a tensile testing machine.

In order to determine a suitable force density function $b_s(t)$, various approaches were tested, which are presented below.

4.2.1 Linear force density function

The first and simplest function used

$$b_s(t) = \dot{b}_0 \cdot t + k \cdot y + k \cdot y_{max} \quad (40)$$

is made up of three terms: The term $\dot{b}_0 \cdot t$ ensures a linear increase in force density over time, as can be seen in Figure 13.

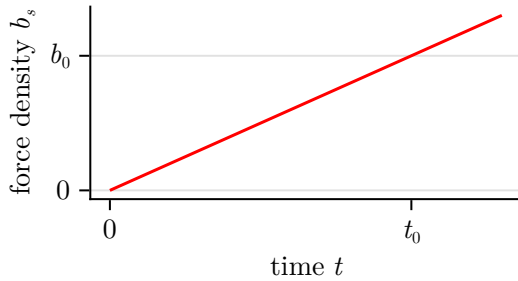


Figure 13 – Schematic curve of force density over time

The term $k \cdot y$ causes the force density to increase linearly over the cross-section. Here k indicates the gradient of the straight line. The greater k is, the greater the difference between the force density acting at the top of the beam cross-section ($y = y_{max}$) and the force density acting at the bottom of the beam cross-section ($y = y_{min}$). The term $k \cdot y_{max}$ with

$$y_{max} = \frac{l_{yz}}{2} = 0.001 \text{ m} \quad (41)$$

ensures that $b_s(t)$ for any k is always positive at the left end of the beam and always negative at the right end of the beam, i.e. the beam is only loaded in compression and not additionally in tension. Otherwise, the

lower part of the beam would be compressed, but the upper part would be pulled apart, which would cause the beam to buckle more quickly, but could not be realized experimentally using a tensile testing machine.

Experiments without the term $k \cdot y_{max}$ also showed that the outermost point layers detached from the beam ends even before the first break (see Figure 14). An attempt was made to circumvent this problem by applying the force not only to the outermost layer, but to a larger area of the beam end. However, this results in more parameters to be set, which can quickly distort the results.

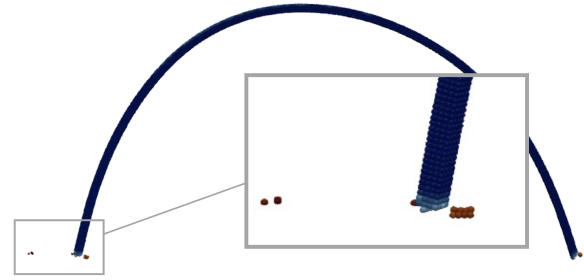


Figure 14 – The outermost point layer detaches (for $\dot{b}_0 = 1.3 \times 10^{10} \text{ N/m}^3 \text{ s}$, $k = 4 \times 10^{12} \text{ N/m}^4$, $t = 0.061 \text{ s}$)

Thus, in addition to the time t with $\dot{b}_0$ (unit $\text{N/m}^3 \text{ s}$) and k (unit N/m^4), there are two parameters that can be varied and influence the fracture behavior. The time course of the Equation 40 as well as the courses of the force density acting at the top ($y = y_{max}$) and bottom ($y = y_{min}$) of the beam, both for the left and right end, are shown in Figure 15.

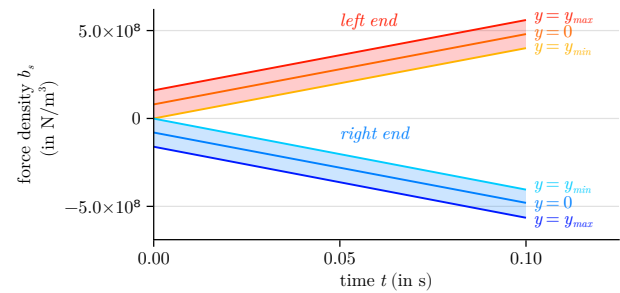


Figure 15 – Linear force density function for $\dot{b}_0 = 4 \times 10^9 \text{ N/m}^3 \text{ s}$ and $k = 8 \times 10^{10} \text{ N/m}^4$

In the following, an attempt is made to determine the parameters in such a way that the breakage of the beam can be simulated as realistically as possible. Ideally, the smallest possible value for k should be selected, because in reality a force that changes over the cross-section of the beam can hardly be realized, especially in the case of a thin spaghetti. Therefore, an attempt is first made

to set $k = 0$ and vary $\dot{b}_0$. The simulation time must be adjusted so that the break can occur within this time. Some results of these simulations are shown in Table 3.

Here, F_E is the force that is applied at the time t_E of the first visible buckling of the beam compared to the rest position.

k [N/m ⁴]	$\dot{b}_0$ [N/m ³ s]	t_E [s]	F_E [N]	t_B [s]	F_B [N]
0	$1 \cdot 10^{12}$	0.0103	20.6	0.0118	23.6
0	1×10^{11}	0.0361	7.2	0.0445	8.9
0	8×10^{10}	0.0396	6.3	0.0489	7.8
0	4×10^{10}	0.0564	4.5	0.0669	5.4
0	2×10^{10}	0.0862	3.4	0.0971	3.9
0	7×10^9	0.1490	2.1	0.1653	2.3
0	4×10^9	0.2027	1.6	0.2235	1.8
0	2×10^9	0.3245	1.3	0.3523	1.4

Table 3 – Variation of $\dot{b}_0$ and its influence on buckling and breaking time as well as buckling and breaking force ($\gamma = 1.0001$)

The specified values for F_E and F_B can be calculated from $\dot{b}_0$ and t_E or t_B .

The following applies to the breaking force according to Equation 36 and Equation 38:

$$F_B = \dot{b}_0 \cdot V_s \cdot t_B. \quad (42)$$

The force F_E acting at the time of buckling can be determined analogously with t_E :

$$F_E = \dot{b}_0 \cdot V_s \cdot t_E. \quad (43)$$

Various findings can be derived from Table 3: The fact that $k = 0$ was chosen means that the beam is first compressed for a long time without buckling. If the values of t_E are compared with those of t_B , it can be seen that approx. 80 to 90% of the time to failure passes without visible buckling of the beam. Accordingly, the buckling of the beam takes place at a very high speed until it breaks. Long simulation times are therefore required to reduce the speed. Furthermore, the direction in which the beam buckles can hardly be influenced. For $\dot{b}_0 = 1 \times 10^{11}$ to 4×10^{10} N/m³s the beam buckles upwards, for $\dot{b}_0 = 2 \times 10^{10}$ to 2×10^9 N/m³s it buckles downwards.

The larger $\dot{b}_0$ is selected, the faster the beam cracks. For very large $\dot{b}_0$ (e.g. $\dot{b}_0 = 1 \times 10^{12}$ N/m³s), the beam changes into a "M form", in which the beam bends and breaks in both directions (see Figure 16).

This is because the beam does not have enough time to bend due to the rapidly increasing force. For $\dot{b}_0 = 1 \times$



Figure 16 – "M-shape" for $\dot{b}_0 = 1 \times 10^{12}$ N/m³s and $k = 0$

10^{11} N/m³s the beam bends, but the cracks do not occur approximately in the middle as expected, but close to the ends of the beam (see Figure 17).

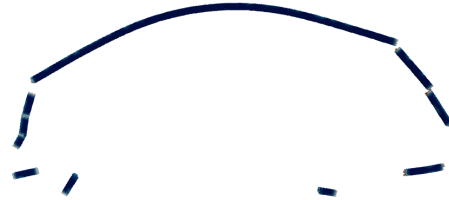


Figure 17 – The fractures occur at the beam ends for $\dot{b}_0 = 1 \times 10^{11}$ N/m³s and $k = 0$

The smaller $\dot{b}_0$ is selected, the more likely the beam breaks in the middle (Figure 18).

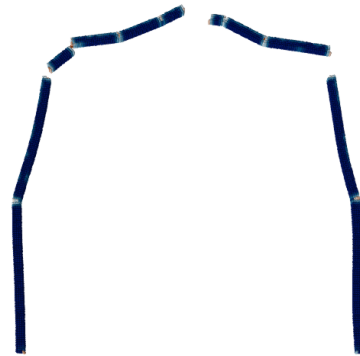


Figure 18 – The breaks tend to occur in the middle of the beam for $\dot{b}_0 = 2 \times 10^9$ N/m³s and $k = 0$

Nevertheless, significantly more simulation time is required to achieve the force leading to fracture. Also, the speed at which the fracture occurs is still so high that fractures into a few fragments are not possible.

It can be observed that F_E and F_B decrease as $\dot{b}_0$ decreases. The reason for this behavior may be the inertia of the beam, which causes the deformation in the area of the beam ends to be greater than with a smaller load due to the large load applied in a short time.

The shorter the time, the stronger this effect becomes. This can also be seen from the results: If $\dot{b}_0$ is doubled from 4×10^{10} to 8×10^{10} N/m³s, F_E increases from 3.4 to 4.5 N, i.e. by 40%. When doubling from 2×10^{10} to 4×10^{10} N/m³s, F_E increases by 32%, when doubling from 2×10^9 to 4×10^9 N/m³s by only 23%. The effect

can also be observed when comparing the differences between F_E and F_B . While for $\dot{b}_0 = 2 \times 10^9 \text{ N/m}^3\text{s}$ there is a difference $F_B - F_E$ of only 0.1 N, the difference for $\dot{b}_0 = 1 \times 10^{11} \text{ N/m}^3\text{s}$ is still 1.7 N. However, other effects could also be the cause of this behavior, since, as already described, the calculated forces are static values, but the beam bending is a dynamic process (with e.g. wave movements).

In the following, a larger value for k must therefore be selected in order to achieve faster buckling of the beam. On the one hand, this should result in a shorter simulation time, which reduces the required computing time. On the other hand, a lower speed at break should be achieved, i.e. simulations with few fragments should also be possible. In Table 4 it is shown how a change in k affects the buckling and fracture time for a fixed $\dot{b}_0$.

k [N/m ⁴]	$\dot{b}_0$ [N/m ³ s]	t_E [s]	t_B [s]
0	4×10^{10}	0.0564	0.0669
8×10^5	4×10^{10}	0.0526	0.0664
4×10^8	4×10^{10}	0.0410	0.0555
8×10^{10}	4×10^{10}	0.0211	0.0383
8×10^{11}	4×10^{10}	0.0067	0.0224

Table 4 – Variation of k and its influence on the buckling and breaking time ($\gamma = 1.0001$)

As can be seen, the break time decreases significantly with increasing k . When comparing t_E with t_B , it can be seen that t_E decreases relative to t_B as k increases. While for $k = 0$ the buckling time is approx. 80 to 90% of t_B , as shown in Table 3, the buckling time decreases to approx. 30% of t_B for $k = 8 \times 10^{11} \text{ N/m}^4$, for example. However, the value for k cannot be increased arbitrarily, otherwise the beam changes into a "M-shape" (Figure 19).



Figure 19 – "M-shape" for $\dot{b}_0 = 4 \times 10^{10} \text{ N/m}^3\text{s}$ and $k = 8 \times 10^{12} \text{ N/m}^4$

A comparison of the results for $k = 0$ with $k = 8 \times 10^{11} \text{ N/m}^4$ is shown in Figure 20. Although the simulation time required is significantly shorter in comparison, the number of fragments has not been reduced. In order to reduce the speed at which the fracture occurs and thus the number of fragments, other values for $\dot{b}_0$ and k must be selected.

The simulation of a fraction in six fragments is shown

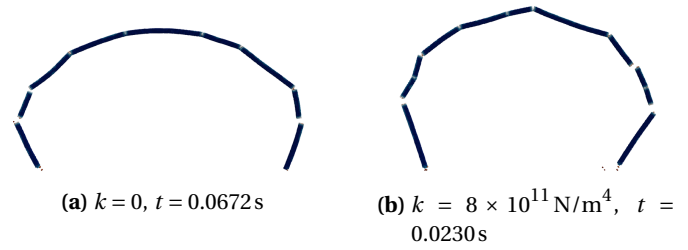


Figure 20 – Comparison of the fracture patterns for different k (with $\dot{b}_0 = 4 \times 10^{10} \text{ N/m}^3\text{s}$)

in Figure 21. The values for $\dot{b}_0$ and k were slightly reduced compared to the previous simulations. Although this increases the simulation time again slightly, it remains within a reasonable range compared to the computational effort.

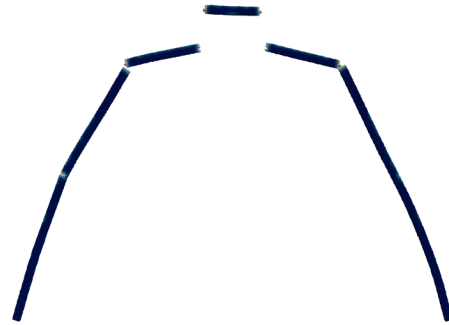


Figure 21 – The beam breaks into six pieces for $\dot{b}_0 = 2 \times 10^9 \text{ N/m}^3\text{s}$ and $k = 7 \times 10^9 \text{ N/m}^4$ ($t = 0.2802 \text{ s}$)

In summary, it can be said that such a linear force density function can be used in principle for the fracture simulation of beams. However, determining suitable values for $\dot{b}_0$ and k is complicated. Fractures in only three or four fragments, as in the four-point bending test, were also not successful. This would probably be possible if $\dot{b}_0$ and k were greatly reduced again, but this would also greatly increase the simulation time. It must also be taken into account that the use of a variable force density across the beam cross-section can hardly be implemented in reality and leads to an immediate, discontinuous increase in force at time $t = 0$.

4.2.2 Sectionally defined force density function

Another approach is to define the force density function section by section and consisting of two straight lines (see Figure 22). This should make it possible to save simulation time by means of a relatively large gradient of the first straight line, as the buckling of the beam is achieved more quickly. At the same time, the speed at which the break occurs can be kept as low as desired by using a low gradient of the second straight line.

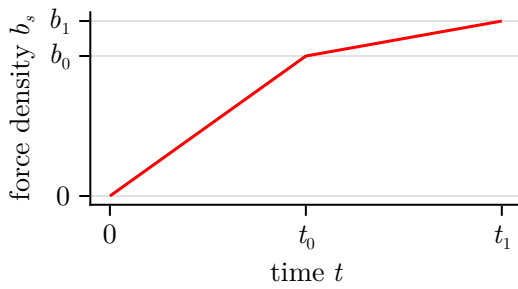


Figure 22 – Schematic curve of the piecewise defined force density function over time

In order to further accelerate the buckling of the beam, a linearly increasing force density is generated across the beam cross-section via the term $k \cdot y$, as in the approaches described above. However, this should only act within the period of the first straight line in order to allow the breakage of the beam caused by the second straight line to occur at the lowest possible speed. The force density curve over time for different areas of the beam is shown in Figure 23.

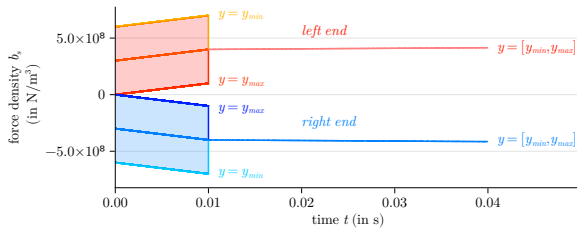


Figure 23 – Piecewise defined force density function for $b_0 = 1 \times 10^8 \text{ N/m}^3$, $k = 3 \times 10^{11} \text{ N/m}^4$, $t_0 = 0.01 \text{ s}$, $b_1 = 1.3 \times 10^8 \text{ N/m}^3$ and $t_1 = 0.07 \text{ s}$

The described function is defined as

$$b_s(t) = \begin{cases} \dot{b}_0 \cdot t + k \cdot y + k \cdot y_{\max}, & t < t_0 \\ \dot{b}_1 \cdot (t - t_0) + b_0, & t > t_0 \end{cases}, \quad (44)$$

where $\dot{b}_0$ is the gradient of the first line and $\dot{b}_1$ is the gradient of the second line. These in turn are defined as

$$\dot{b}_0 = \frac{b_0}{t_0} \quad \text{and} \quad \dot{b}_1 = \frac{b_1 - b_0}{t_1 - t_0}. \quad (45)$$

This results in five parameters to be varied: b_0 , b_1 , t_0 , t_1 and k . In order to define the function in the range $t = 0$ to $t = t_0$, the corresponding parameters were selected so that the beam buckles as much as possible in the shortest possible time without breaking. The assumed values are listed in Table 5.

In order to define the slope of the second straight line, an attempt was made to achieve the lowest possi-

b_0	t_0	k
$[\text{N/m}^3]$	$[\text{s}]$	$[\text{N/m}^4]$
1×10^8	0.01	3×10^{11}

Table 5 – Defined values of the first straight line

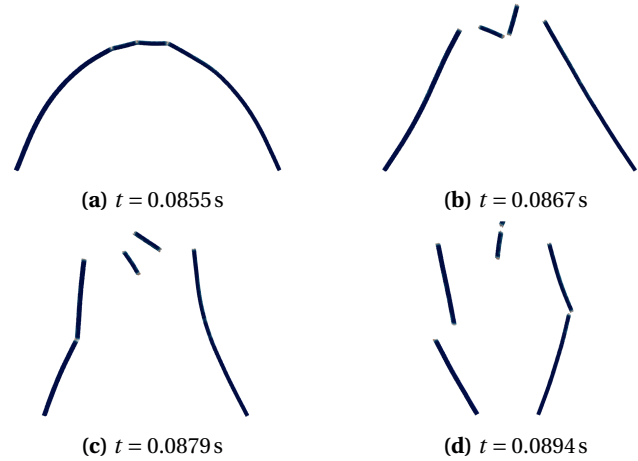


Figure 24 – Longer after the break into four parts, two further breaks take place (for $b_0 = 1.6 \times 10^8 \text{ N/m}^3$)

ble speed of the break with a reasonably long simulation time. Some simulation results are listed in Table 6, which show the effect of different straight line slopes on the number of fragments and simulation time.

t_1	b_1	fragments	t_B
$[\text{s}]$	$[\text{N/m}^3]$		$[\text{s}]$
0.07	1.1×10^8	5	0.2111
0.07	1.3×10^8	5	0.1002
0.07	1.6×10^8	4	0.0852
0.07	2×10^8	6	0.0758
0.07	3×10^8	7	0.0646
0.07	10×10^8	8	0.0456

Table 6 – Variation of b_1 and its effect on the number of fragments and fracture time ($\gamma = 1.08$)

As the results show, a smaller slope of the second straight line causes a break into fewer fragments, as expected. It must be mentioned that for $b_0 = 1.6 \times 10^8 \text{ N/m}^3$ (see Figure 24) for example, the longer fragments begin to oscillate strongly and break again a relatively long time after the actual break into four fragments. The beam therefore does not actually break into four fragments, but into six.

However, as the simulation time for all the simulations

carried out in this work only extends to a short time after the first fracture and it can never be ruled out that fractures will still occur after this time and that more fragments will therefore be created, the fragments created due to the oscillation were neglected. Furthermore, it can be stated that the beam buckles almost immediately. This approach therefore leads to very short buckling times compared to the linear force density function already described, which has a positive effect on the simulation times and thus the computational effort.

In summary, it can be said that this approach reduces the simulation time. It is also possible to reduce the number of fragments if the fractures caused by the oscillation of long beam sections are neglected. A disadvantage, however, are the many parameters to be set, which make the simulation complicated.

4.3 Velocity-controlled load

Instead of a force density, the beam is now subjected to a velocity $v(t)$ at both ends. This acts on the lowest point layer ($y = y_{min}$), as shown in Figure 25.

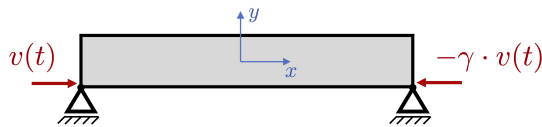


Figure 25 – Loading the ends of the beam with a velocity function

The function $v(t)$ thereby

$$v(t) = \begin{cases} \frac{v_0}{t_0} \cdot t, & t < t_0 \\ v_0, & t > t_0 \end{cases} \quad (46)$$

is defined piecewise in two straight lines, as also shown in Figure 26. As with the approaches already described, this is intended to achieve a rapid buckling of the beam with subsequent break at low speed.

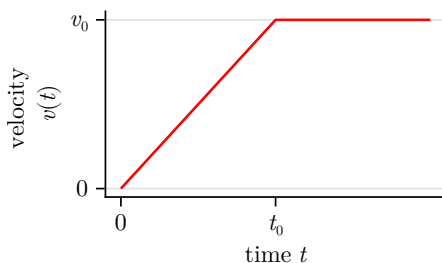


Figure 26 – Schematic progression of the piecewise defined velocity function

With this approach, two parameters must be set with v_0 and t_0 . Some simulation results for a fixed value of

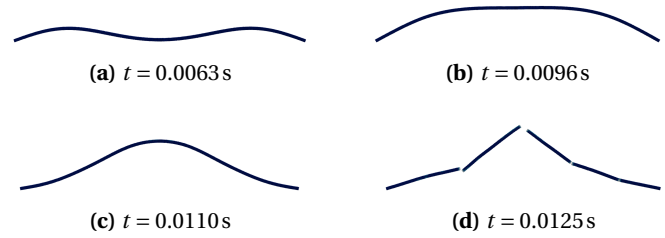


Figure 27 – Strong swinging of the beam with subsequent breakage into five parts (for $v_0 = 1$ m/s, $t_0 = 0.01$ s)

t_0 and different values of v_0 are listed in Table 7 (s_B here denotes the distance by which the beam must be compressed to break).

v_0 [m/s]	fragments	t_B [s]	s_B [m]	oscillations
1	5	0.0110	0.0060	1
0.9	4	0.0110	0.0054	1
0.8	4	0.0111	0.0049	1
0.7	4	0.0113	0.0044	1
0.6	4	0.0172	0.0073	2
0.5	3	0.0184	0.0067	2
0.4	2	0.0298	0.0099	4
0.3	3	0.0546	0.0149	9
0.2	4	0.1112	0.0212	20
0.1	5	0.3368	0.0332	>20

Table 7 – Variation of v_0 and its effect on the number of fragments and fracture time ($t_0 = 0.01$ s, $\gamma = 1.1$)

Based on the fracture pattern, the results can essentially be divided into three ranges. For $v_0 = 1$ to 0.7 m/s, the beam changes from the rest position to a "M-shape" (Figure 27 a) due to the high velocity, then bends in one direction (b and c) and breaks into several parts (d). If the speed is reduced to $v_0 = 0.6$ to 0.4 m/s, the beam also bends into a "M-shape" and then in one direction. However, this phenomenon is repeated at least once before the beam breaks. The more often the phenomenon is repeated, the fewer parts the beam breaks into. It is noteworthy that the beam for $v_0 = 0.4$ m/s, after the oscillation has been repeated three times in the described manner, only breaks into two parts (Figure 28). For $v_0 = 0.3$ to 0.1 m/s the number of oscillations increases significantly. The number of beam fragments also increases. For $v_0 = 0.1$ m/s the frequency of the oscillations is so high that they are barely recognizable (see Figure 29). The beam tends to break in the same way as in the previous approaches.

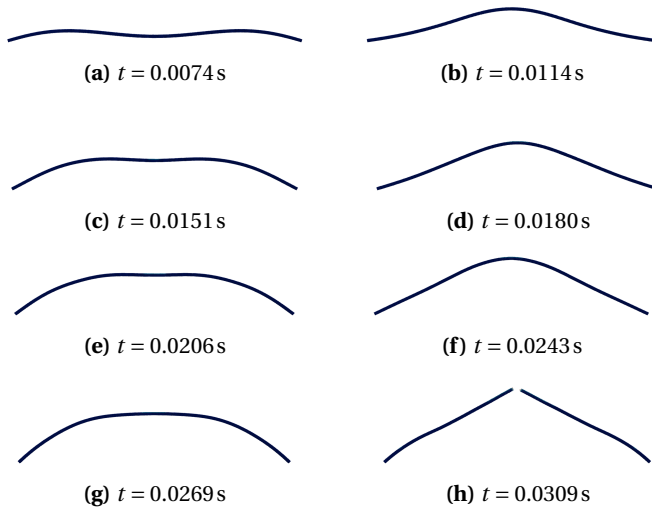


Figure 28 – The beam breaks into only two parts (for $v_0 = 0.4 \text{ m/s}$, $t_0 = 0.01 \text{ s}$)

If the velocity curve is integrated, the distance by which the beam is compressed until it breaks can be determined:

$$s_B = v_0 \left(t_B - \frac{1}{2} t_0 \right). \quad (47)$$

Theoretically, the beam would always have to be compressed by approximately the same distance to break, regardless of the speed. However, the distance increases with decreasing v_0 in Table 7, which is probably due to the described oscillation phenomenon.

Pre-bent beam

In order to avoid the described vibration problem due to the strong buckling of the beam and to reduce the simulation time, simulations of a beam that is already slightly buckled in one direction are carried out below (see Figure 30). The beam is loaded with the velocity function in Equation 46. The unstressed beam shape at

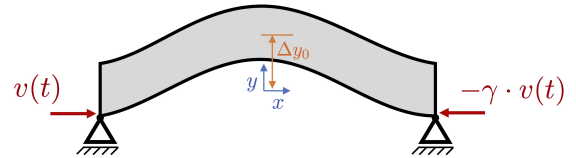


Figure 30 – Loading the bent beam with a velocity function

time $t = 0$ corresponds to the function curve of

$$y(x) = -\frac{4\Delta y_0}{l_x^2} x^2 + \Delta y_0 \quad (48)$$

for all $x \in \left[-\frac{l_x}{2}, \frac{l_x}{2}\right]$. The factor Δy_0 indicates the deflection of the beam.

Basically, the approach described here causes two problems: The pre-buckled beam is stress-free at the time $t = 0$ of the simulation. As a result, this beam buckles further until failure than a comparable non-pre-buckled beam, which results in unrealistic failure behavior. An example of a strongly pre-bent beam is shown in Figure 31. Another problem is the beam ends, which in the model used here are always parallel to the y -axis despite deflection. The beam geometry is therefore modeled incorrectly (see Figure 32). The smaller Δy_0 is selected accordingly, the less strongly the effects described come into play. In order to find a good compromise between sufficiently realistic fracture behavior and short simulation time, the value was set to

$$\Delta y_0 = 30\Delta x = 0.015 \text{ m}. \quad (49)$$

Assuming this value, simulation results are shown in Table 8 are shown for different values of v_0 , which are

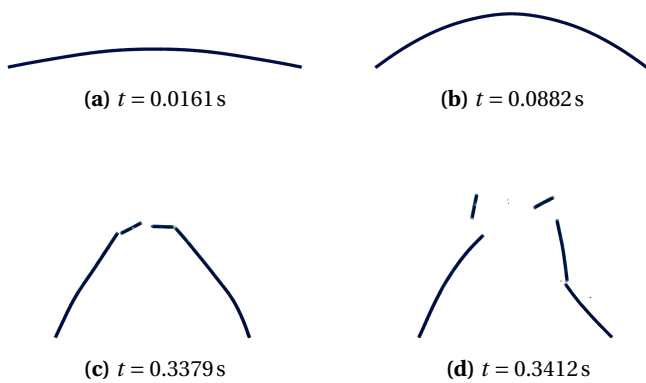


Figure 29 – The beam breaks into five fragments (for $v_0 = 0.1 \text{ m/s}$, $t_0 = 0.01 \text{ s}$)

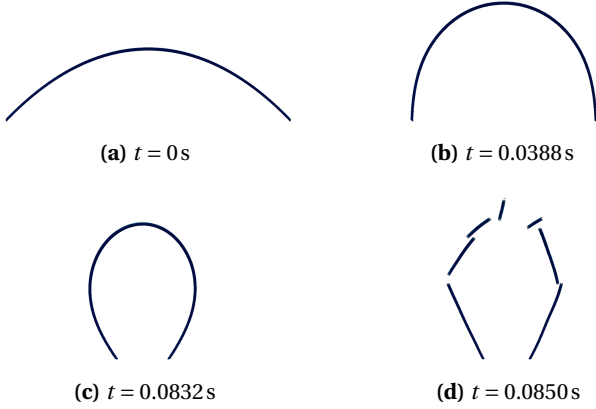


Figure 31 – Fraction of a beam pre-bent with $\Delta y_0 = 100\Delta x = 0.05$ m

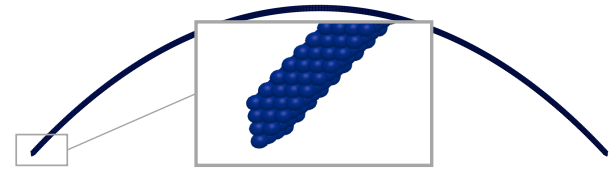


Figure 32 – "Cut-off" end of beam for $\Delta y_0 = 0.05$ m

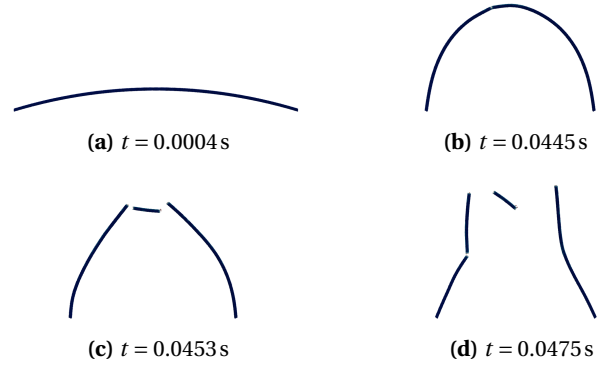


Figure 33 – Breakage of the pre-bent beam into four fragments (for $\Delta y_0 = 0.015$ m, $v_0 = 1$ m/s)

Δy_0 [m]	v_0 [m/s]	fragments	t_B [s]	s_B [m]
0.015	7	5	0.0106	0.0392
0.015	6	5	0.0111	0.0366
0.015	5	4	0.0115	0.0325
0.015	4	6	0.0158	0.0342
0.015	3	4	0.0168	0.0354
0.015	2	4	0.0249	0.0398
0.015	1	4	0.0442	0.0392
0.015	0.7	4	0.0594	0.0381
0.015	0.5	4	0.0839	0.0395
0.015	0.3	6	0.1326	0.0383
0.015	0.1	6	0.3916	0.0387

Table 8 – Variation of v_0 and its influence on the fracture behavior of the pre-buckled beam ($t_0 = 0.01$ s, $\gamma = 1.1$)

discussed below. For the range $v_0 = 7$ to 3 m/s, the behavior of the beam already described in the previous chapter, which is dominated by strong vibrations, is no longer visible from a speed of $v_0 = 2$ m/s. In the range $v_0 = 2$ to 0.1 m/s, the behavior of the beam is therefore more likely to be as expected according to the experimentally determined fracture patterns, as shown for $v_0 = 1$ m/s in [Figure 33](#). Furthermore, it can be seen that from a velocity of $v_0 = 0.5$ m/s the simulation time increases significantly.

Using [Equation 47](#), the distance s_B by which the beam is compressed until it breaks can be calculated. The calculated values for s_B are also listed in [Table 8](#) and show that the path is always between 3 and 4 cm. In contrast to the results of the non-pre-buckled beam in the previous chapter, the path is therefore always approximately the same, which is an improvement on the previous approach. Another advantage resulting from this fact is that the required simulation time can be easily calculated by transforming [Equation 47](#) as:

$$t_B = \frac{s}{v_0} + \frac{1}{2} t_0. \quad (50)$$

In most of the other approaches presented in this paper, however, the simulation time cannot simply be calculated, but it must be found out by trial and error whether the breakage of the beam takes place in the assumed simulation time or not.

5 Summary

The aim of this work, to simulatively investigate the influence of various parameters and boundary conditions on the fracture behavior of long beams, was achieved. Furthermore, a sensible alternative to the existing four-point bending test was developed.

The results of the simulations using the four-point bending test showed that the fracture behavior to be expected from tests with spaghetti can be reflected in the simulations.

However, the disadvantage is the many parameters to be set, which can quickly distort the results.

Although the approach of applying different cross-section-dependent force density functions to the beam ends reduces the number of parameters, the fracture behavior that occurred in the simulations is highly sensitive to the simulation parameters. The simulation results deviate from the results of the experimental tests for small changes in the parameters (k and $\dot{b}_0$).

In addition, the simulation time required is sometimes significantly longer. The simulation time can be reduced by introducing a force density that varies over the cross-section of the beam. However, this cross-section-dependent force density can hardly be realized experimentally.

The subsequently developed approach of loading the beam ends with a velocity function also reduces the number of parameters. The simulation time can be kept sufficiently low via the deflection of the pre-buckled beam, even if this also potentially distorts the results. Furthermore, when using this method, it is possible to simply calculate the required simulation time before carrying out the simulation, which is why it does not have to be determined by trial and error. In contrast to the method described in the previous paragraph, beam breaks could be simulated in similarly few fragments as when using the four-point boundary condition could be simulated.

In summary, it can be said that the velocity boundary condition developed in this work provides a relatively easy-to-apply approach that can be used for subsequent systematic peridynamic investigations of fracture and damage in thin beams.

Code Availability: All simulations described in this article were performed using the Peridynamics.jl library, which can be found at

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All data and code are available upon request from the author.

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