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Fundamental Climate Dynamics in Energy Balance Models: Stability, Paleoclimate Extremes, and Greenhouse Gas Forcing

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Abstract: *This study investigates fundamental mechanisms of the Earth's climate system using energy balance models (EBMs) of varying complexity. A zero dimensional (0D), globally averaged model and a one dimensional (1D), latitudinally resolved model were employed to address the following research questions: (i) How stable is the climate system under external perturbations? (ii) How can the* Faint Sun Paradox *be examined within a simplified modeling framework? (iii) How does the climate respond to increasing concentrations of greenhouse gases? and (iv) To what extent do 0D and 1D EBMs differ in their ability to represent these phenomena? Both models are capable of reproducing key features of the climate system, including feedback mechanisms such as the ice albedo effect. The 1D model provides enhanced spatial resolution and more detailed representations of latitudinal processes, whereas the 0D model exhibits greater sensitivity to parametric changes. Simulations indicate that the climate system can exhibit strong nonlinear responses, including abrupt transitions into globally glaciated states, known as* Snowball Earth *scenarios. These results highlight the potential and the limitations of low complexity climate models in capturing essential system behavior. The comparison between model types underscores the trade off between computational simplicity and physical detail, informing model selection for conceptual and exploratory climate studies.*

Keywords: Energy Balance Models, Climate Dynamics, Snowball Earth, Climate Sensitivity, Multiple Equilibria

1 Introduction

Climate change represents one of the greatest global challenges of our time. Despite decades of efforts to reduce emissions, a new record of 41.6 gigatons of global CO_2 emissions was reached in 2024 [7]. At the same time, atmospheric CO_2 concentrations have reached levels not observed for several hundred thousand years [14]. In a world projected to have nearly 10 billion people by 2050 [29], energy consumption, resource demands, and habitat requirements are also increasing. The consequences of anthropogenic influence on the climate system are already visible today in the form of melting sea ice, increasing extreme weather events, and climate-induced migration. Events such as the wildfires in California [9], floods in Pakistan [22], or droughts in East Africa [15] underscore the urgency with which these developments must be addressed.

The prediction and understanding of climatic changes require scientific methods that adequately capture the highly complex nature of the Earth system. Climate models represent a central analytical tool that allows for the systematic investigation of feedback processes, inertias, and nonlinearities. Modern Earth System Models (ESMs) integrate various physical, chemical, and biological processes in coupled subsystems and provide high-resolution projections at both global and regional scales. Despite their capabilities, ESMs come with significant drawbacks: high computational demand, limited transparency, and restricted potential for analytical insight into the underlying mechanisms.

Already in the 1960s and 1970s, the first simple climate models, so called energy balance models (EBMs), were developed to mathematically describe fundamental processes of the energy balance and ice-albedo feedback [3, 27]. At the same time, Syukuro Manabe developed the first Global Circulation Models (GCMs), which featured significantly higher complexity and spatial resolution, forming the basis for modern Earth system models [19]. However, EBMs were not designed as a deliberate counterpoint to these complex models, but rather served as complementary, theoretically oriented tools to investigate individual mechanisms in an abstracted form.

The role of such models has not diminished since then, on the contrary, in current climate research, EBMs serve as important theoretical tools to specifically investigate feedbacks, parameter spaces, and sensitivities. The use of simplified models is not seen as opposed to complex Earth system models, but rather as a complementary approach that isolates and generalizes key mechanisms [11]. This model-theoretical duality enables an adequate understanding of both the depth and breadth of climatic processes. This perspective on the importance of energy balance models, despite their reduced complexity, is also supported by Paeth [23], who emphasizes their usefulness for sensitivity studies and process understanding.

Spiegl et al.[28] clearly demonstrated this complementarity by employing the Planet Simulator (PlaSim), an Earth System Model of intermediate complexity, conducting 37 sensitivity experiments testing key climate parameters within a single model framework. Their results show that even with a structurally simplified model, important feedbacks and climatic thresholds can be reliably represented. This approach facilitates systematic parameter comparisons and helps to identify critical internal model interactions essential for realistically simulating extreme climate states such as the Neoproterozoic Snowball Earth. Thus, simplified EBMs remain indispensable tools for isolating and generalizing fundamental mechanisms alongside high-complexity models.

A key aspect within modeling is the so called climate sensitivity, defined as the equilibrium warming following a doubling of the atmospheric CO_2 concentration. Current estimates of the sensitivity to a CO_2 doubling range between 2.5 and 4 °C [14], with uncertainties primarily attributed to feedback processes. EBMs provide a particularly suitable means to isolate and quantify the influence of such processes in a modeling framework. Furthermore, EBMs can also address questions regarding the existence and nature of multiple equilibrium states, a topic that is gaining increasing relevance in light of potential tipping points within the climate system.

An additional perspective is provided by the investigation of paleoclimatic scenarios. Climate epochs such as the Cryogenian with global glaciation [10, 16] or the Faint Sun Paradox [26] present natural extreme conditions under which feedback processes manifested in particularly pronounced ways. While the quantitative reconstruction of these epochs requires the use of specialized paleoclimate models, qualitative analysis using EBMs allows for a deeper understanding of fundamental mechanisms. The historical perspective thus serves as a physical framework for evaluating current sensitivities and potential system instabilities. It highlights that many processes debated today were already active in Earth's history, albeit under different boundary conditions.

The aim of this work is therefore to investigate key questions regarding the stability of the climate system using both, own developed and adapted energy balance models. The focus is on three complementary lines of inquiry: First, the bistability of the climate is analyzed, i.e., the question under which conditions the system can occupy two different stable equilibrium states. Second, sensitivity analyses are conducted to determine how sensitive the global temperature level is to changes in greenhouse gas concentrations. Third, paleoclimatic extreme states serve as test scenarios to reflect the explanatory power of simplified models under exceptional boundary conditions and to identify and explain abstract mechanisms of the Earth system.

By deliberately choosing an abstract modeling framework, this work aims to provide a scientifically grounded contribution to the understanding of the internal logic of climatic processes, independent of short-term forecasting goals but with high relevance for the long term assessment of climate risks.

2 Methods

2.1 Energy Balance Equation

The foundation of the climate model is the energy balance equation, which calculates the difference between incoming and outgoing radiation:

$$\Delta E = Q_{in} - Q_{out} \tag{1}$$

The incoming energy

$$Q_{in} = (1-\alpha)\cdot\frac{S}{4} \tag{2}$$

indicates how much thermal radiation is absorbed by the Earth. Here, the solar constant $S = 1365\,\mathrm{W/m^2}$ represents the amount of energy hitting a surface perpendicular to the sun on Earth. The solar constant S is divided by 4 because the incoming radiation is distributed from the circular area (πr^2) to the entire spherical surface ($4\pi r^2$). α denotes the albedo and defines the reflection of solar radiation that reaches the Earth. The model uses a temperature dependent albedo, which will be discussed in more detail later.

The term for outgoing energy describes the thermal radiation emitted by the Earth into space:

$$Q_{out} = \sigma\cdot\varepsilon(T)\cdot T^4 \tag{3}$$

The Stefan-Boltzmann constant σ indicates how much an ideal black body radiates at a given temperature. Since the Stefan-Boltzmann law $\sigma\cdot T^4$ only describes the radiation of an ideal black body, the emission coefficient ε is introduced.

2.2 Temperature Dependent Effective Emissivity

In this model, the **effective emissivity** ε_{eff} is used instead of a constant value. It varies with surface temperature T to account for the influence of atmospheric conditions on outgoing longwave radiation. The nonlinear dependence of water vapor on temperature is not explicitly implemented in this 0D and 1D energy balance model. Instead, a piecewise linear approximation within empirically constrained bounds is adopted.

The effective emissivity is defined piecewise using the cold state emissivity ε_{cold}, the hot state emissivity ε_{hot}, the reference transmissivity τ_0, and the temperature sensitivity parameter β:

$$\varepsilon_{eff}(T) = \begin{cases} \varepsilon_{cold}, & T \le T_{ref} - 10\,^\circ\mathrm{C}, \\ \tau_0\left(1-\beta(T-T_{ref})\right), & |T-T_{ref}| < 10\,^\circ\mathrm{C}, \\ \varepsilon_{hot}, & T \ge T_{ref} + 10\,^\circ\mathrm{C}. \end{cases} \tag{4}$$

with

- $\varepsilon_{cold} = 0.64$,
- $\varepsilon_{hot} = 0.58$,
- $\tau_0 = 0.612$,
- $T_{ref} = 14\,^\circ\mathrm{C}$,
- $\beta = 0.004\,\mathrm{K^{-1}}$.

Pragmatic justification of the bounds and slope. The cap values $\varepsilon_{cold} = 0.64$ and $\varepsilon_{hot} = 0.58$ ensure that the effective emissivity remains physically plausible for extreme temperatures. Within the ±10 °C range around T_{ref}, the linear slope β was chosen so that the effective emissivity smoothly interpolates between these capped values. This provides a simple, temperature dependent adjustment that improves model realism compared to a constant emissivity. While this approach is not an exact physical representation, it is a pragmatic solution that enhances the accuracy of the model in a straightforward and implementable way.

2.3 Thermal Inertia

The Earth's temperature change is influenced not only by the net energy imbalance but also by the amount of energy required to heat the atmosphere. This depends on the heat capacity C_p and density ρ of the atmospheric layer. In this model, the effective heat capacity is assumed to be equal to that of the entire atmospheric column above one square meter of Earth's surface, which is approximately $10^7\,\mathrm{J/(m^2\cdot K)}$. This value is taken to be equivalent to the heat capacity of about 2.5 meters of ocean water. By making this assumption, the model incorporates thermal inertia, allowing for a delayed climate response to changes in energy balance.

2.4 Incorporation of Greenhouse Gases

One of the most important components in the climate model is accounting for the influence of greenhouse gases on the climate. For this purpose, the following relation was used [21]:

$$k_{CO_2}\cdot\ln\left(\frac{CO_2}{CO_{2,0}}\right) + k_{CH_4}\cdot\ln\left(\frac{CH_4}{CH_{4,0}}\right) + k_{N_2O}\cdot\ln\left(\frac{N_2O}{N_{2O,0}}\right) \tag{5}$$

This formula includes the greenhouse gases with the greatest impact on the climate: carbon dioxide (CO_2), methane (CH_4), and nitrous oxide (N_2O). The influence of the greenhouse gases is represented logarithmically in this equation. This reflects the behavior of the Earth's atmosphere: initially, an increase in greenhouse gases has a very strong effect on radiative forcing, but this effect diminishes at higher concentrations.

In the equation, current values of greenhouse gas concentrations are divided by pre industrial reference values to precisely determine the influence of human activities. The pre industrial greenhouse gas concentrations used as references are [18]:

$$CO_{2,0} = 280\,\mathrm{ppm}, \quad CH_{4,0} = 750\,\mathrm{ppb}, \quad N_{2O,0} = 270\,\mathrm{ppb}$$

This allows for a clear identification of the extent of human influence. Furthermore, radiative forcing coefficients $k_{CO_2} = 5.35\,\mathrm{W/m^2}$, $k_{CH_4} = 0.036\,\mathrm{W/m^2}$, and

$k_{N_2O} = 0.12\,\mathrm{W/m^2}$ are used [21], which were empirically determined to accurately simulate the individual effects of greenhouse gases on atmospheric forcing. This entire equation is subtracted from the Q_{out} equation in the model, as greenhouse gases reduce the amount of thermal radiation escaping into space.

2.5 Numerics

Numerics is a central topic in the programming of the model. First, it is important to explain what numerical methods are and why they play a role in the model. Numerical methods are used to compute approximate solutions for differential equations. This is applied in the model because the equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\Delta E}{C_p \cdot \rho} \tag{6}$$

is time dependent, as follows from the energy balance introduced in Section 2.1.

There are various numerical methods. Initially, the forward Euler method was used. This method is relatively simple and stable enough for initial purposes. In the forward Euler method, the temperature derivative is used to determine the next temperature value. The basic formula is:

$$T_{n+1} = T_n + \Delta t \cdot \frac{\Delta E(T_n)}{C_p \cdot \rho} \tag{7}$$

Here, T_n is the current temperature, Δt is the length of the time step, and $\Delta E(T_n)$ is the energy balance at the previous temperature. Initially, there were no problems with this method. However, as the model complexity increased, it became less stable, and the time step had to be kept very small.

For this reason, the more stable and accurate Runge Kutta 4 method was adopted. Unlike the Euler method, it calculates several derivatives at different points within the time step:

$$\begin{aligned}
k_1 &= \Delta E(T_n) \\
k_2 &= \Delta E\left(T_n + \frac{\Delta t}{2} \cdot \frac{k_1}{C_p \cdot \rho}\right) \\
k_3 &= \Delta E\left(T_n + \frac{\Delta t}{2} \cdot \frac{k_2}{C_p \cdot \rho}\right) \\
k_4 &= \Delta E\left(T_n + \Delta t \cdot \frac{k_3}{C_p \cdot \rho}\right)
\end{aligned}$$

The new temperature value is computed by a weighted sum of these derivatives:

$$T_{n+1} = T_n + \frac{\Delta t}{6} \cdot \frac{k_1 + 2k_2 + 2k_3 + k_4}{C_p \cdot \rho} \tag{8}$$

Since the Runge Kutta method calculates the slope multiple times during a time step and considers new intermediate points, the slope is continuously adjusted. This results in a more accurate approximation and a more stable solution [17, 25].

2.6 Temperature Dependent Albedo

The albedo effect is a crucial factor in climate modeling and significantly influences the climate system. While many standard models use a fixed, averaged albedo, this model employs a dynamic albedo that varies continuously as a function of temperature. This allows for a more realistic response of the model to extreme climate conditions.

The albedo values range between a minimum of approximately 0.295 for ice-free surfaces and a maximum of 0.70 for a fully ice covered Earth (Snowball Earth). The function uses a smooth transition based on the hyperbolic tangent (`tanh`) to determine the albedo value depending on temperature.

The parameter T_{center} defines the temperature at which the transition occurs. The parameter δ controls the steepness of the transition: the smaller δ is, the sharper the change between low and high albedo values.

Mathematically, the albedo $\alpha(T)$ is calculated as:

$$\alpha(T) = \alpha_{\min} + \frac{\alpha_{\max} - \alpha_{\min}}{2} \times \left(1 + \tanh\left(\frac{T_{\text{center}} - T}{\delta}\right)\right) \tag{9}$$

This means that at temperatures well below T_{center}, the albedo approaches the maximum value (ice-covered surface), while at high temperatures, it approaches the minimum albedo (ice-free surface). Between these extremes, the albedo varies smoothly and physically plausibly.

This function enables a flexible and realistic modeling of the albedo effect, especially for scenarios involving strong temperature fluctuations and ice formation.

2.7 Introduction to the Climate Model

In addition to the custom climate model, the more complex one-dimensional energy balance model by Wagner and Eisenman [32] was employed. This model enables simulation of additional climate system components such as temperature variations across latitudes and changes in sea ice extent, allowing improved validation and contextualization of results.

Several modifications were implemented to tailor the model to specific research questions. Greenhouse gases CO_2, CH_4, and N_2O were incorporated, as they were not originally included.

A central improvement addresses the treatment of outgoing longwave radiation. The original model approximates it linearly as

$$Q_{\text{out}} = A + BT,$$

which is computationally efficient but lacks accuracy for extreme climate states. To improve this, a correction step was introduced that replaces the linear approximation with a physically grounded Stefan-Boltzmann emission term, incorporating a temperature-dependent effective emissivity $\varepsilon(T)$:

$$Q_{\text{SB}} = \varepsilon(T)\,\sigma\,T^4,$$

where σ is the Stefan-Boltzmann constant.

To maintain numerical stability and consistency with the original model formulation, the corrected solar forcing term is calculated as

$$Q'_{\text{solar}} = Q_{\text{solar}} - Q_{\text{SB}} + (A + BT),$$

This formulation effectively replaces the original linearized outgoing radiation with a more accurate Stefan Boltzmann based flux.

Additionally, the model uses the Berger and Loutre [1] [2] insolation, which provides a more detailed and realistic temporal variation of solar forcing compared to the simplified version used previously. For the implementation, the method of Huybers and Eisenman [13] is used.

Together, these enhancements increase the model's fidelity in simulating both current and extreme climate regimes. Additional modifications were also made, which can be examined directly in the code.

The computational workflow of the modified 1D EBM is summarized in Figure 1.

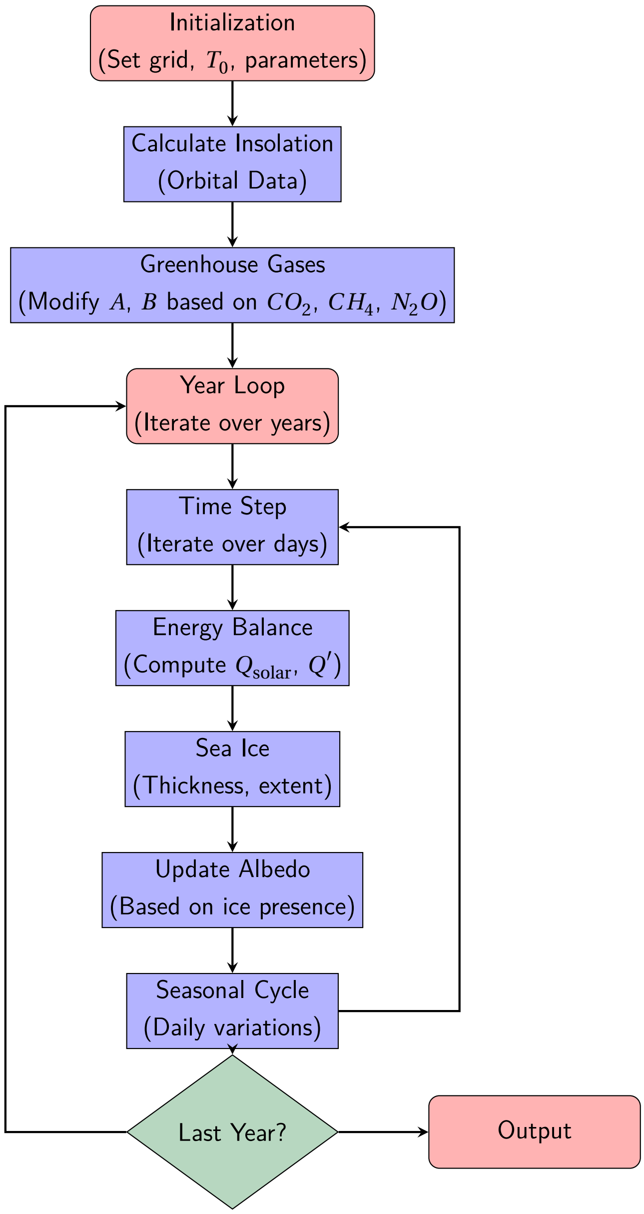

Figure 1 – Flowchart of the 1D model showing the main steps: Initialization, Insolation, Greenhouse Gases, Energy Balance, Sea Ice, Albedo, Seasonal Cycle, and Outputs.

2.8 Experiments with the Climate Model

Experiment 1: Turning Off the Sun **Objective:** The experiment investigated the climate's response to a X-year long interruption of solar energy.

Implementation: The sun was turned off for a period between 1 and 10 years by setting the solar constant in the model to zero. Example from the Python program:

```
for years in range(dur):
    if 20 <= years < x:
        # Turn off solar radiation
        current_S = 0
    else:
        # Restore normal solar radiation
        current_S = S_initial
```

Research Questions:

- Could the climate recover after a period without solar energy?
- If not, to which equilibrium did the climate shift?

Experiment 2: Reduction of Solar Energy (Faint Sun Paradox) **Objective:** The experiment simulated conditions in early Earth history, when solar energy was 30% weaker. The goal was to understand why Earth did not become completely glaciated despite the reduced radiative input.

Implementation: The solar constant was scaled to simulate the reduced solar energy:

```
current_S = S_initial * 0.7
```

Research Questions:

- Did the reduced solar energy lead to a Snowball Earth scenario?
- What concentration of greenhouse gases was necessary to prevent this scenario?

Experiment 3: Sensitivity Studies with Greenhouse Gases **Objective:** The experiment investigated the climate system's sensitivity to altered concentrations of CO_2. The goal was to determine the impact of increased CO_2 concentrations on the climate.
Implementation:

```
current_CO2 = CO2_initial * 2
```

Research Questions:

- How did the climate change when CO_2 was doubled?

3 Results

3.1 Snowball Earth

Before presenting the results of the Snowball Earth and bistability experiment, a brief introduction to the plot structure of the climate model will be given. This will be used for interpreting the experiments below, without revisiting the individual plots in detail each time. For this purpose, the model was run with preindustrial greenhouse gas levels and present day solar radiation. In this scenario, a "typical preindustrial" climate behavior was achieved, where temperatures near the equator remain around 25–30°C year-round and decrease significantly towards the poles (see Figure 2).

Experiment 1 In the first experiment, as described in Section 2.8, the Sun was switched off for x years after the spin up phase. All other parameters remained unchanged. This experiment aimed to investigate whether and when the climate might shift into a different state under such conditions. It was observed that already after just one year without solar radiation, the system transitioned into a Snowball Earth (see Figure 3).

Thus, the experiment indicates bistability: after the perturbation, the system does not return to the preindustrial equilibrium but remains in a Snowball Earth state. The Figure 3 clearly shows that temperatures on Earth, unlike the example in Figure 2 have dropped significantly. In winter, temperatures in the polar regions reach as low as -80°C, while in summer they rise close to 0°C at the pole and -10°C at the equator. The high ice thicknesses at low latitudes are also noteworthy.

The reason why the climate system collapses into a Snowball Earth even after just one year without solar radiation is primarily due to the ice albedo runaway feedback. This is a positive feedback mechanism in which the initial cooling caused by the absence of solar radiation promotes further ice formation. As ice coverage increases, the surface albedo rises, which (under normal solar conditions) reduces solar energy absorption and causes further cooling. Although no solar radiation is present during one year, this feedback effect continues to maintain and reinforce the cold state beyond the initial year, preventing the system from recovering. Consequently, the Earth increasingly moves into a negative energy balance and can eventually enter a Snowball Earth state from which it cannot easily recover. However, if temperatures remain above the critical thresholds, ice does form during the colder seasons, but the ice albedo effect is not strong enough to trigger a runaway process. This allows the radiation balance to stabilize again, and temperature values return to the typical preindustrial levels without further climatic influences.

The 0D model was also tested to observe the effects of turning off the solar radiation for a year. As shown in Figure 4, the mean temperature decreases significantly more slowly compared to the 1D model. This slower decline is primarily due to the 0D EBM's dependence on the albedo parameterization, which caps the maximum albedo at 0.7, reflecting its globally averaged nature. Nevertheless, the 0D model similarly enters a persistent Snowball Earth state when solar radiation is completely removed. On average, the 0D model produces slightly colder global temperatures than the 1D model, stabilizing around -43°C, whereas the 1D EBM stabilizes near -38°C. The close agreement between the two models highlights the robustness and functionality of the simpler 0D approach.

Remarks on the Snowball Earth The Snowball Earth experiment demonstrated the sensitivity of the climate equilibrium to external forcings. Specifically, turning off the sun for a prolonged period showed that the system can rapidly transition into a fully glaciated state. It became evident that once certain thresholds are crossed, the climate can become stably locked in this Snowball Earth state due to strong feedback mechanisms.

3.2 Experiment 2: Faint Sun Paradox

The following experiment investigates the Faint Sun Paradox. It analyzes how the climate would theoretically look with a 30% weaker sun without any other influences. The resulting climate state is shown in Figure 5.

The results show that a new equilibrium forms, which clearly differs from the Snowball Earth results. Similarly, the 0D model also exhibits a colder stable climate state, with a global mean temperature settling at -60°C. So the temperatures in the 0D model in the faint sun scenario are also significantly colder than those in the Snowball Earth as can be seen in comparison to Figure 4. In the 1D model, global temperatures are even lower. During winter, the equatorial regions remain the warmest areas at approximately -70°C, with temperatures decreasing progressively toward the poles. In summer, however, there are hardly any differences between the different latitudes: temperatures are around -35°C everywhere. Interestingly, the low latitudes are slightly colder in summer than the higher latitudes.

The ice thickness follows an expected distribution: in the equatorial regions, it is about 26 m and increases towards the poles. At the pole, it reaches a maximum thickness of approximately 31.5 m. The seasonal ice thickness cycle shows hardly any changes, with the ice thickness remaining constant throughout the year. The ice edge behaves similarly, showing no shifts in this state.

This new equilibrium exhibits even colder temperatures than the Snowball Earth states from experiment one. The question arises as to how the Earth could still have had a mild climate like the pre industrial period under such circumstances. Various factors may have influenced the climate. The hypothesis of Carl Sagan, one of the discoverers of the Faint Sun Paradox, is examined here. Sagan suspected that the Earth's atmosphere in the past had a different composition, specifically with a higher concentration of greenhouse gases [26].

In the 1D model, reproducing the pre industrial state shown in Figure 2 would require an unrealistically high increase in CO_2 levels, by a factor of 20 to 30 thousand compared to pre industrial values. This is likely due to the logarithmic relationship between CO_2 concentration and radiative forcing, which causes the impact of additional CO_2 to diminish at higher concentrations.

3.3 Experiment 3: CO_2 Doubling

The final experiment was conducted to establish a modern connection to current research. The question investigated was what happens to the climate system when CO_2 is doubled. For the analysis, two different models

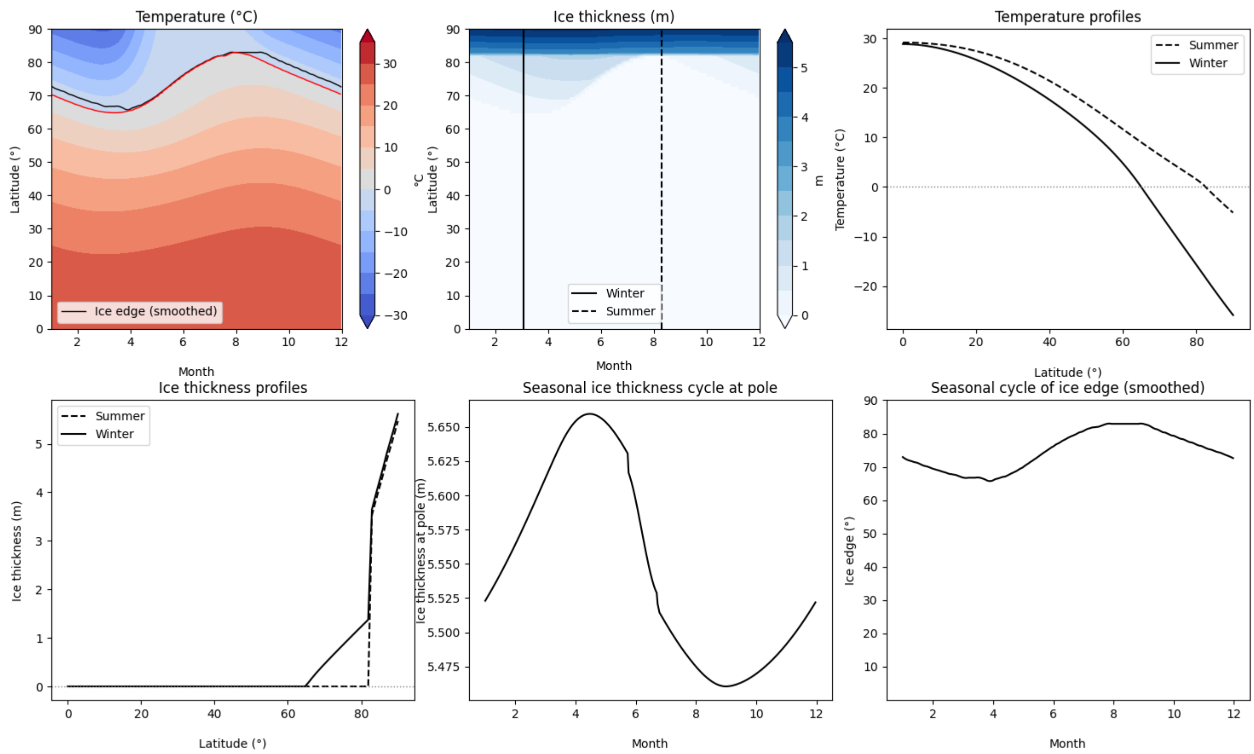

Figure 2 – Six subplots each depicting the climate behavior in the final (500th) simulation year. The top left plot uses color contours to show the temperature distribution over months (x-axis) and latitudes (y-axis); the black line indicates the ice edge. Top center: ice thickness, with two vertical lines marking winter and summer points. Top right plot: temperature profiles for summer and winter. The bottom left plot shows the distribution of ice thickness across latitudes. The middle bottom plot displays the seasonal variation of ice thickness at the pole, and the bottom right subplot illustrates the annual progression of the ice edge across latitudes.

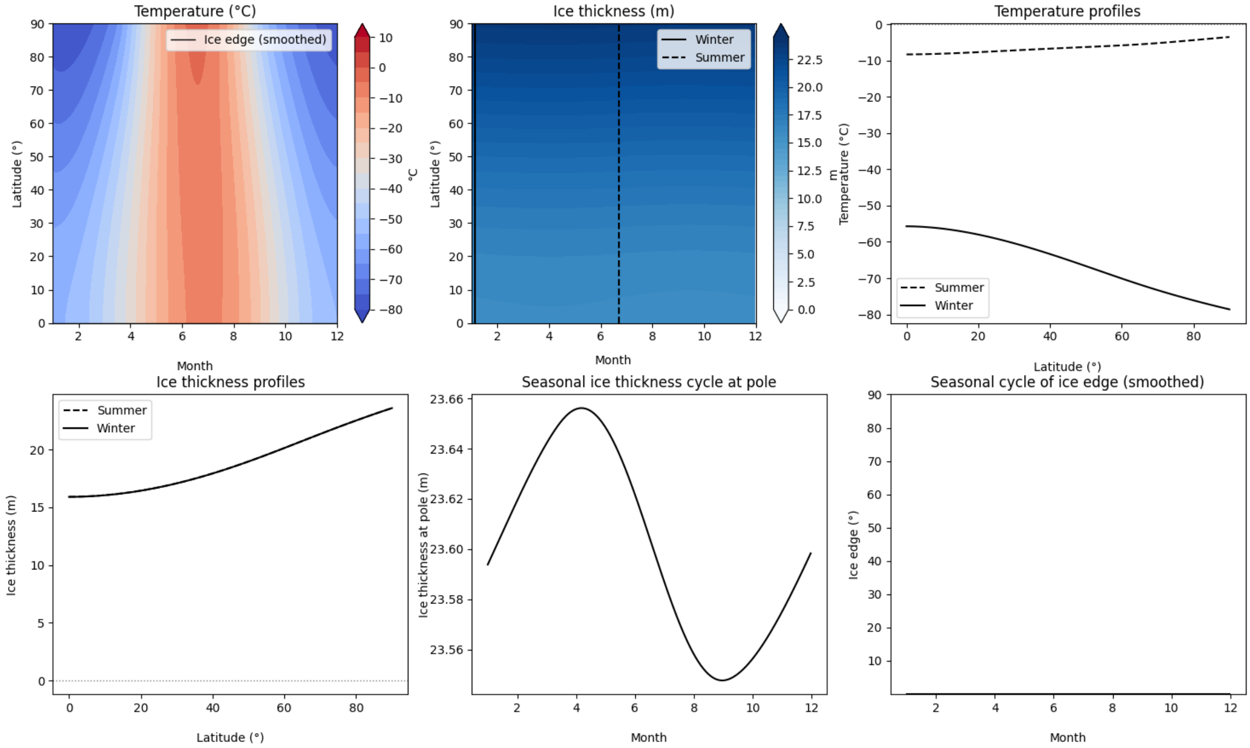

Figure 3 – Climate behavior in the final simulation year after the transition into a Snowball Earth state. The panels show seasonal temperature, ice thickness, summer and winter temperature profiles, latitudinal ice-thickness distribution, polar ice thickness variation, and annual ice edge evolution.

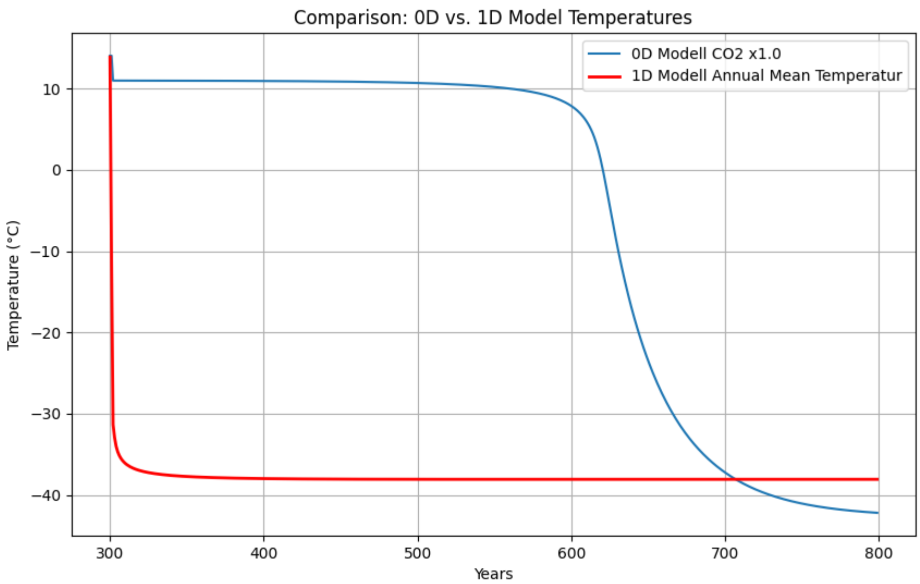

Figure 4 – Evolution of the globally averaged temperature after the spin up in the 0D and 1D model illustrating the transition into a Snowball Earth state.

were used: on the one hand, the global temperature development was calculated with the 0D model, and on the other hand, the modified model that had already been used in the previous experiments.

Figure 6 shows the global temperature development at different CO_2 concentrations with the preindustrial baseline value of 280 ppm. The blue curve in the graph depicts the temperature development at 420 ppm from the 0D model, which corresponds to the CO_2 level in the atmosphere in the year 2023. For examining the climate

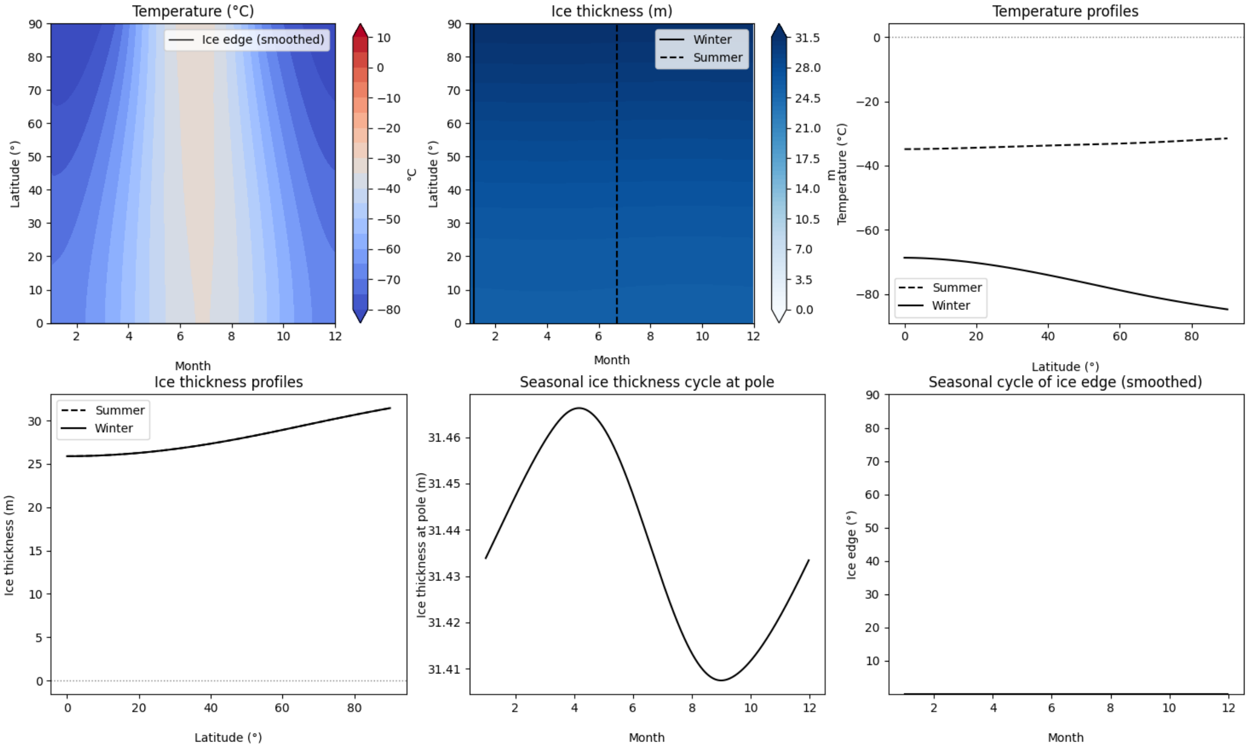

Figure 5 – Climate behavior in the final simulation year under 30% reduced solar irradiance. The panels show seasonal temperature, ice thickness, summer and winter temperature profiles, latitudinal ice thickness distribution, polar ice thickness variation, and annual ice edge evolution.

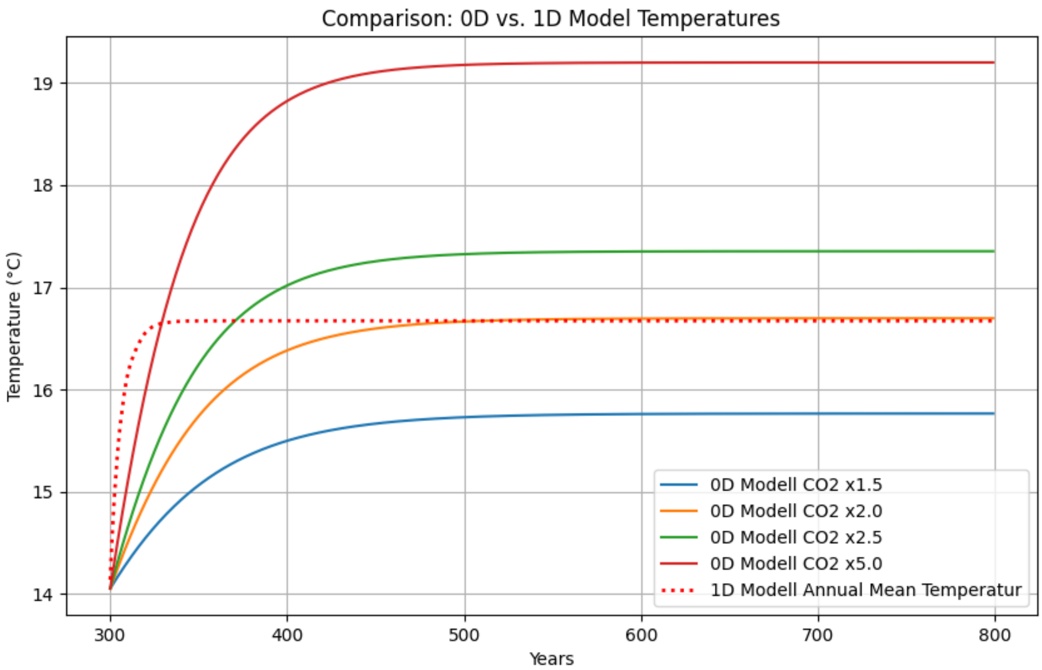

Figure 6 – Global temperature development at different CO_2 concentrations in the 0D and 1D model.

sensitivity to a doubling of CO_2, the orange curve is particularly relevant, showing that the temperature rises by about 2.6 °C in such a scenario. The other curves are shown for rough orientation of the climate sensitivity. Here, the logarithmic behavior of the greenhouse gas effect is clearly visible. The red dashed line in the plot shows the global mean temperature increase from the 1D model for the CO_2 doubling scenario. The temperature increases by about 2.6 °C, which is same as increase from the 0D model.

The doubling of the CO_2 concentration also leads to significant changes in the climate system compared to the preindustrial state (Figure 2), as shown in Figure 7. While temperatures in the low latitudes range between 25 and 32 degrees, the polar regions warm considerably more. Compared to Figure 2, it is evident that the ice boundary has shifted toward the poles. In the mid latitudes, no ice is now present even during the coldest phase. The ice thickness has also decreased compared to Figure 2. The maximum ice thickness in the polar regions is now only about 3.75 meters.

Explanation: The sensitivity study shows that a doubling of CO_2 has a substantial impact on the climate. The general temperature increase can be explained by the greenhouse effect: CO_2 absorbs the infrared radiation emitted by the Earth's surface and re-emits part of it back into the atmosphere. This causes the atmosphere to warm. The logarithmic behavior of greenhouse gases, including CO_2, is easily explained. CO_2 absorbs thermal radiation in a specific wavelength range, particularly in the 15-micrometer band. At high concentrations of CO_2, this absorption band becomes increasingly saturated, meaning that the radiation in this wavelength range is already largely absorbed. Additional CO_2 molecules therefore have a weaker effect.

The 0D model predicts a global temperature increase of about 2.6 degrees for a doubling of CO_2. Similarly, the 1D model also predicts an approximate temperature increase of 2.6 degrees for the same scenario. However, by resolving temperature changes across latitudes, the 1D model captures spatial feedbacks such as the albedo effect from ice melt, which leads to amplified warming in the polar regions. This effect, known as polar amplification, influences regional temperatures but results in a comparable global mean temperature increase to that suggested by the 0D model.

4 Discussion

Snowball Earth

The results of the Snowball Earth experiment agree in many aspects with those found in the literature, but also show some deviations. The investigations demonstrate that even a relatively short interruption of solar radiation can lead to a completely new climate equilibrium. This behavior is in line with the concept of bifurcation points described in studies on Snowball Earth dynamics [30, 12]. The central mechanism, also discussed extensively in the results section, is the *ice-albedo runaway* effect, which is frequently cited in the literature.

In contrast to most studies, which typically simulate a gradual reduction of solar irradiance such as Voigt et al.[31], who reduced solar input by up to 6% the present study applies a complete shutdown of solar radiation for one year. Such extreme scenarios are rarely investigated in the literature, where solar dimming experiments are far more common. In the experiments of Voigt et al.[31], the coupled atmosphere ocean general circulation model (AOGCM) required approximately 355 years to fully transition into a Snowball Earth state, whereas their nonlinear 0D energy balance model reached this state in around 264 years. This is consistent with the general trend that more complex, coupled GCMs due to their explicit representation of ocean heat transport, circulation dynamics, and atmospheric feedbacks exhibit longer transition times than simpler EBMs. Interestingly, in the present work, the opposite behavior is observed: the transition time of the 0D EBM is longer than in the corresponding 1D model. This deviation is likely linked to differences in model formulation, particularly the albedo parameterization. In my 0D EBM, the albedo function is globally averaged and capped at a maximum value of 0.7, which constrains the system's reflectivity compared to the 1D model, where a more spatially resolved albedo parameterization can attain higher values in fully ice-covered states. As a result, the positive ice albedo feedback is somewhat weaker in my 0D EBM, slowing the approach to a fully glaciated equilibrium.

Overall, while the results align qualitatively with established Snowball Earth dynamics, the differences in transition times highlight the strong dependence on model structure and experimental design. Direct comparisons with studies such as Voigt et al.[31] should be treated with caution, as variations in dimensionality, forcing scenarios, and parameterizations can substantially influence the outcomes. Consequently, the findings here should be interpreted as illustrating fundamental mechanisms and sensitivities rather than providing precise quantitative predictions for the real climate system.

Faint Sun Paradox

The investigations into the Faint Sun Paradox yield two central findings: first, the extremely cold climate states with a 30% reduced solar irradiance without additional influences; and second, the exceptionally high concentrations of greenhouse gases required to simulate a preindustrial climate.

In the first scenario, a stable state with extreme cold was observed, featuring average winter temperatures around $-75\,^{\circ}$C and summer values near $-35\,^{\circ}$C. As most previous studies adopt a 20% reduction in the solar constant, an additional simulation with this forcing was

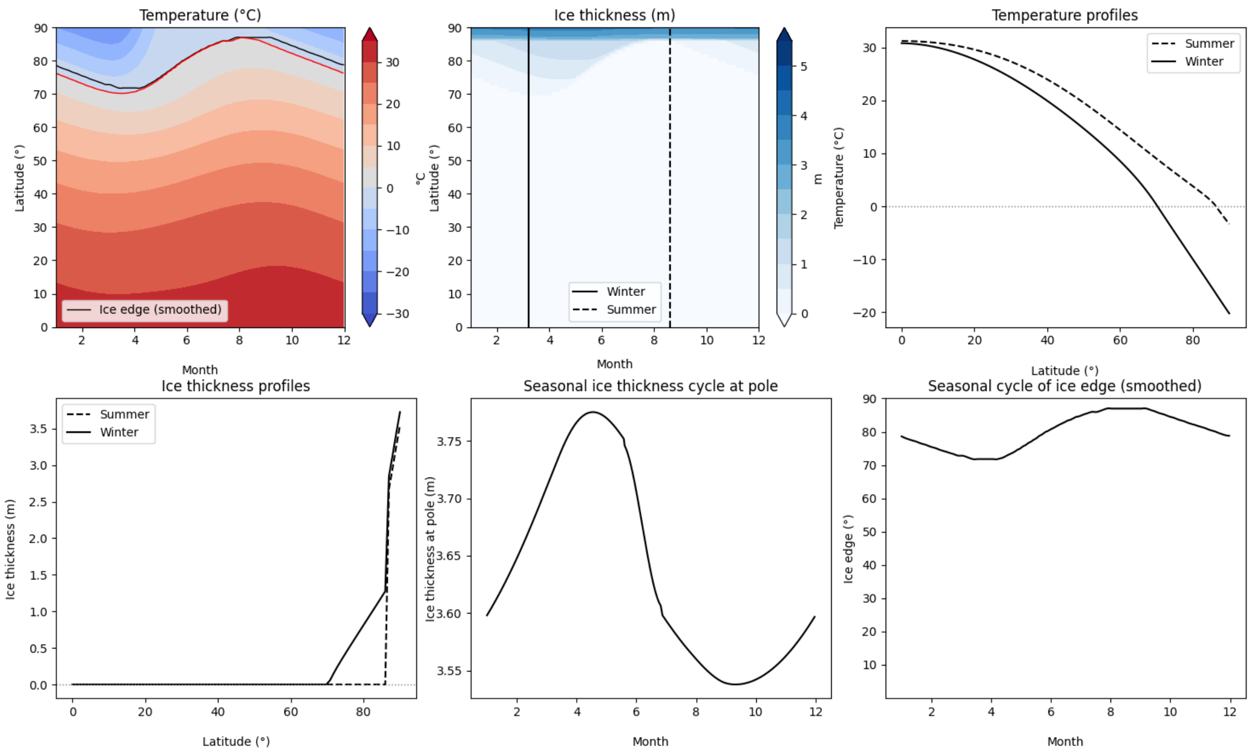

Figure 7 – Climate behavior in the final simulation year for doubled CO_2 concentrations. The panels show seasonal temperature, ice thickness, summer and winter temperature profiles, latitudinal ice thickness distribution, polar ice thickness variation, and annual ice edge evolution.

performed for comparison. This yielded a global annual mean temperature of approximately −49 °C. In contrast, Charnay et al.[5] utilizing a 3D Global Climate Model (GCM) found a mean of −54 °C under the same insolation reduction. The slightly higher temperatures in our simulation are likely attributable to the differences in model complexity. The 1D Energy Balance Model (EBM) that is used in this study employs simplified albedo and heat transport parameterizations and lacks full atmospheric and oceanic dynamics, which tend to amplify cooling in more sophisticated GCM setups.

However, deviating from the literature, the required greenhouse gas concentration to reconstruct a preindustrial climate appears significantly higher: the results indicate a necessary increase by a factor of 20–30 thousand for CO_2. Converted, this corresponds to a CO_2 partial pressure of about 7 bar. The reason for this discrepancy likely lies in the model structure. Gérard, Hauglustaine, and François [8] also required a 20,000-fold CO_2 concentration to prevent a global glaciation of the Earth at 30% reduced solar luminosity using a 1.5D EBM model. This discrepancy is partly due to the absence of other climate feedback mechanisms, such as cloud effects, in the energy balance model used here. As noted by Byrne and Goldblatt [4], models lacking these feedbacks tend to overestimate the forcing required from increased CO_2 to avoid global glaciation under reduced solar luminosity.

Sensitivity Studies with CO_2

The sensitivity studies show a global temperature increase of approximately 2.6 °C for a doubling of CO_2 concentration. This value lies at the lower end of the equilibrium climate sensitivity (ECS), which according to Intergovernmental Panel on Climate Change (IPCC)[14] ranges between 2.5 and 4 °C. Further estimates, such as those by Cox, Huntingford, and Williamson[6], yielded a value of about 2.8 °C and are based on a relationship between short term temperature variability and long-term climate sensitivity. These results confirm that the models used adequately represent the fundamental processes of CO_2 climate sensitivity.

Another observed effect is the disproportionately strong warming in the polar regions, accompanied by a significant shift of the ice edge and an almost complete retreat of ice during the summer months. This phenomenon, known as *polar amplification*, is also described in the literature [20] and is explained in the model by the albedo feedback effects. Particularly noteworthy is the logarithmic effect of CO_2 on temperature development. This occurs because CO_2 absorbs thermal radiation in specific wavelength bands, which become increasingly saturated with higher concentrations. As a result, additional CO_2 molecules contribute only marginally to further warming. This relationship is clearly evident in the results and is also confirmed in studies such as Plass[24].

The results indicate that the model captures the primary processes driving CO_2 induced warming, but the absence of higher order feedbacks such as dynamic cloud responses or coupled ocean atmosphere interactions means that its ECS estimate should be interpreted as a lower bound approximation.

Comparison of 0D and 1D Energy Balance Models

Both 0D and 1D energy balance models (EBMs) enable the investigation of fundamental climate mechanisms and feedbacks, such as the ice-albedo effect. Both model types exhibit similar basic climate responses to forcings, including the stability of climate states. However, they differ significantly in their sensitivity to model parameters and modifications.

In the 0D model, even small adjustments, such as changes to the albedo function, can have large impacts on the results, potentially causing key feedbacks like the onset of a Snowball Earth state to appear or vanish. This makes the 0D model highly sensitive to parameter details.

In contrast, the 1D model is more robust against minor changes, with larger modifications required to alter fundamental climate mechanisms or to induce alternative equilibrium states. This increased stability arises from the model's spatial resolution and the latitudinal energy exchange, which represent additional physical processes that buffer the climate response against small parameter variations.

Societal Context and Relevance

The results obtained are not only scientifically significant but also socially relevant. Climate models play a central role in political decision-making and public discourse on climate change. The self developed model used in this study offers a high degree of transparency and enables a fundamental understanding of climatic processes, an advantage over more complex models with opaque structures. This accessibility also promotes a broader societal understanding of the physical foundations of climate change.

The sensitivity analyses clearly demonstrate how strongly the climate system responds to a doubling of CO_2 concentrations. The logarithmic relationship in particular highlights the necessity of early climate mitigation measures, as later reductions are only marginally effective.

Furthermore, the study draws attention to the existence of climatic tipping points, especially in connection with the bistability of the climate system. Although no tipping points were simulated in the model for rising temperatures, it nevertheless illustrates the dynamics of such transitions. Real systems, such as the Greenland and Antarctic ice sheets, are examples of potential tipping elements where irreversible melting could occur. The results presented thus provide important indications of how quickly such processes may be initiated.

Overall, the study contributes not only to the scientific understanding of climatic interrelationships but also builds a bridge between science, society, and politics. It illustrates how a deeper understanding of the climate system is essential for sustainable development, both to prevent severe consequences and to foster an informed societal discourse.

5 Conclusions and Outlook

The aim of this study was to investigate fundamental processes and mechanisms of the climate system using zero and one dimensional EBMs. The central questions were: Does bistability exist in the climate system and how does it manifest? How can the Faint Sun Paradox be explained? And how sensitive is the climate system to changes in greenhouse gas concentrations?

The model simulations conducted confirm the existence of bistability in the climate system. Only one year of turning off the sun led to clearly distinguishable equilibrium states and revealed potential tipping points. In this scenario, surfaces remained ice covered throughout the year, with global winter temperatures dropping to $-80\,^{\circ}\mathrm{C}$ and summer values close to $0\,^{\circ}\mathrm{C}$.

For the investigation of the Faint Sun Paradox, the one dimensional EBM was used. To compensate for the 30% reduction in solar radiation, significantly elevated greenhouse gas concentrations were applied. The resulting equilibrium climate corresponds to pre industrial conditions. However, the exact magnitude of the required greenhouse effect varies depending on the model; the literature reports significantly lower values for a comparable stabilization. Without additional greenhouse-gas forcing, a new stable climate equilibrium was established, which is considerably colder than the classical Snowball Earth state observed in the other experiments.

With regard to climate sensitivity, the model simulations yielded a warming of approximately $2.6\,^{\circ}\mathrm{C}$ for a doubling of CO_2 concentration, a value that falls within the range of current scientific estimates. In addition, a significantly stronger warming in polar regions was observed, an effect consistent with the mechanism of polar amplification. At the same time, a substantial decline in ice cover in the mid latitudes, as well as a significant reduction in ice thickness, was recorded. These results highlight the high sensitivity of the climate system to changes in greenhouse gas concentrations. The results obtained in this study demonstrate that even simplified models, such as energy balance models, are capable of realistically capturing key climate feedbacks, tipping points, and sensitivities. The depth of insight gained illustrates the relevance of such models for a systematic understanding of climatic processes and risks.

In future work, the developed model will be further expanded. A key step will be the integration of geographical aspects, particularly the specific distribution of land and ocean surfaces. The climatic impact differs significantly depending on whether, for example, landmasses are located over the equator or not a factor that strongly influences both heat storage and feedback processes such as the ice albedo interaction.

In addition, the aim is to engage with more complex models in the future, such as General Circulation Models (GCMs). The goal is to gain a deeper understanding of climate relevant mechanisms by comparing them with simplified energy balance models and to investigate their interactions with more detailed and realistic system processes. The parallel application and alignment of different model types offer a promising opportunity to combine analytical insights with high predictive accuracy.

The long term goal remains to enable a deeper, scientifically grounded understanding of the dynamics of climate change through model based approaches both for the research community and for a broader public audience.

Code Availability: The source code supporting the results of this study is publicly available on GitHub in the accompanying repository: access the repository.

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